Introduction to Search

CSI 4106

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Preamble

Message of the Day

Too much social media gives AI chatbots 'brain rot', Nature News, 2025-10-31.

Learning Objectives

- **Describe** the role of search algorithms in AI, crucial for planning, reasoning, and applications like AlphaGo.
- **Recall** key search concepts: state space, initial/goal states, actions, transition models, cost functions.
- **Identify** the differences of uninformed search algorithms (BFS and DFS).
- **Implement** BFS and DFS and **compare** them using the 8-Puzzle problem.
- **Analyze** performance and optimality of various search algorithms.

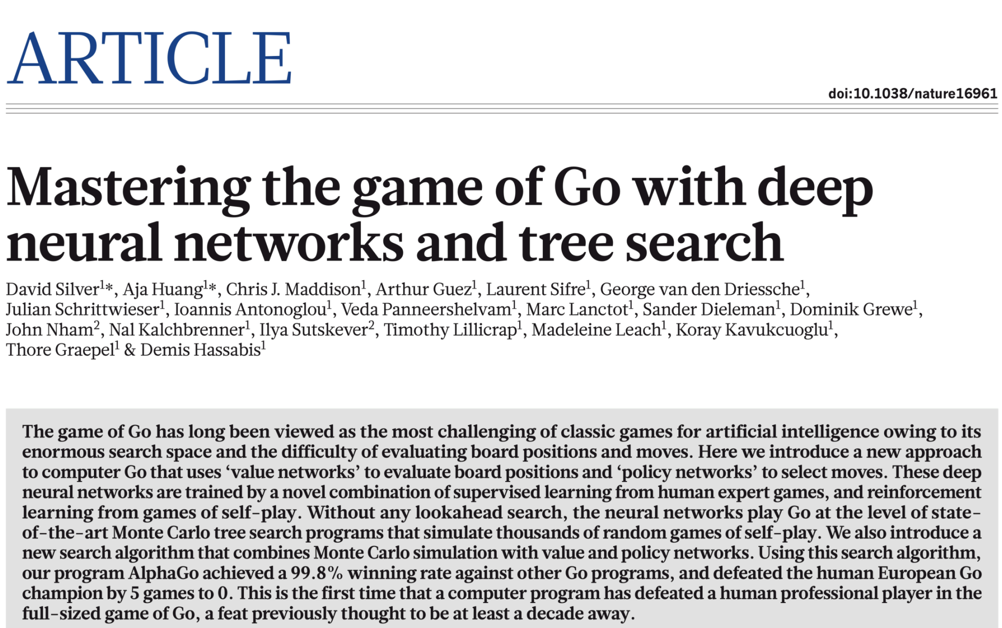
The main objective of this presentation is to justify the role of search methods in artificial intelligence, as well as to introduce the terminology for upcoming courses. You already have knowledge of breadth-first and depth-first search algorithms. We will leverage this expertise to establish the necessary vocabulary for future courses and to demonstrate the need to develop more “intelligent” algorithms. We will study the following themes.

- Uninformed Search
- Informed Search
- Local Search
- Population-Based Metaheuristics
- Adversarial Search
- Monte Carlo Tree Search

This will provide us with a solid foundation to approach our final segment dedicated to formal reasoning.

Justification

Justification



ARTICLE

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Mastering the game of Go with deep neural networks and tree search

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The game of Go has long been viewed as the most challenging of classic games for artificial intelligence owing to its enormous search space and the difficulty of evaluating board positions and moves. Here we introduce a new approach to computer Go that uses ‘value networks’ to evaluate board positions and ‘policy networks’ to select moves. These deep neural networks are trained by a novel combination of supervised learning from human expert games, and reinforcement learning from games of self-play. Without any lookahead search, the neural networks play Go at the level of state-of-the-art Monte Carlo tree search programs that simulate thousands of random games of self-play. We also introduce a new search algorithm that combines Monte Carlo simulation with value and policy networks. Using this search algorithm, our program AlphaGo achieved a 99.8% winning rate against other Go programs, and defeated the human European Go champion by 5 games to 0. This is the first time that a computer program has defeated a human professional player in the full-sized game of Go, a feat previously thought to be at least a decade away.

Silver et al. (2016)

Justification

We have honed our expertise in **machine learning** to a point where we possess a robust understanding of **neural networks** and **deep learning**, allowing us to develop simple models using **Keras**.

. . .

In recent years, **Monte Carlo Tree Search (MCTS)** has played a pivotal role in advancing **artificial intelligence** research. After initially concentrating on deep learning, **we are now shifting our focus to search**.

Justification

The integration of **deep learning** and **MCTS** underpins modern applications such as **AlphaGo**, **AlphaZero**, and **MuZero**.

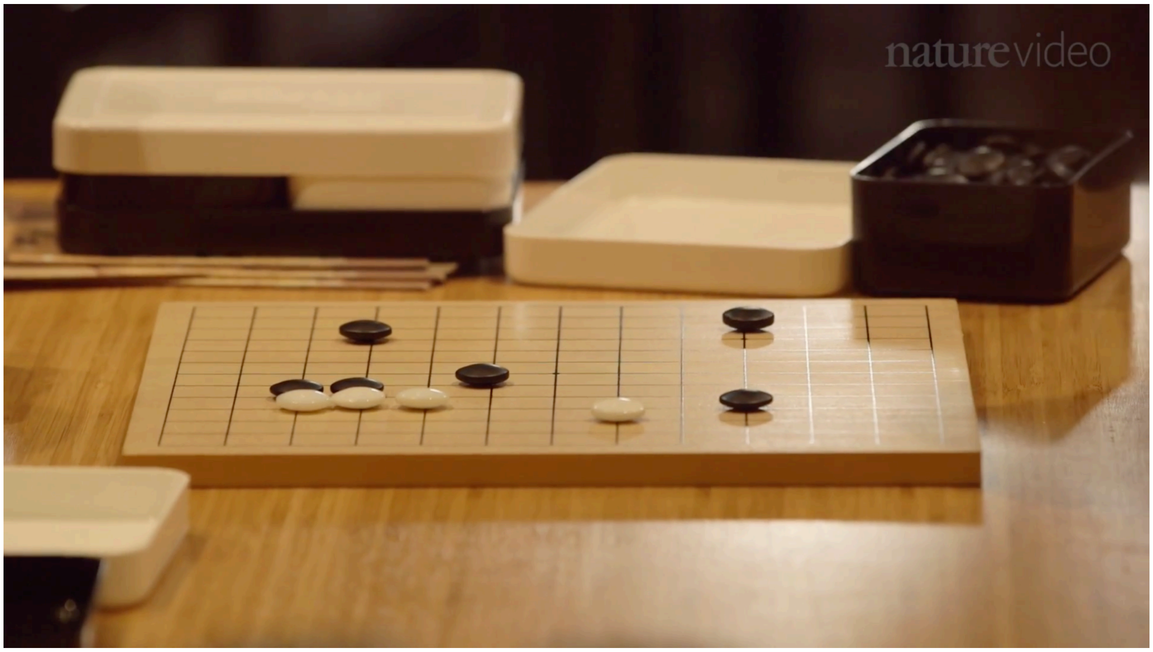
. . .

Search algorithms are crucial in addressing challenges in **planning** and **reasoning** and are likely to become **increasingly significant in future developments**.

Search (a biased timeline)

- **1968 – A***: Heuristic-based search, foundational for pathfinding and planning in AI.
- **1970s-1980s – Population-Based Algorithms** (e.g., Genetic Algorithms): Stochastic optimization approaches useful for large, complex search spaces.
- **1980s – Constraint Satisfaction Problems (CSPs)**: Search in structured spaces with explicit constraints; a precursor to formal problem-solving systems.
- **2013 – DQN**: Reinforcement learning via search (Q-learning) from raw input (pixels).
- **2015 – AlphaGo**: Game-tree search with Monte Carlo Tree Search (MCTS) combined with deep learning.
- **2017 – AlphaZero**: Generalized self-play with MCTS in multiple domains.
- **2019 – MuZero**: Search in unknown environments without predefined models.
- **2020 – Agent57**: Generalized search across multiple environments (Atari games).
- **2020 – AlphaGeometry**: Search-based theorem proving in mathematical spaces.
- **2021 – FunSearch**: Potentially another generalization of search techniques.

Search



The computer that mastered Go. Nature Video posted on YouTube on 2016-01-27. (7**m** 52**s**)

In 1997, IBM’s Deep Blue defeated the reigning world chess champion, Garry Kasparov. However, the AI community was not particularly impressed, as the system’s primary accomplishment lay in its ability to evaluate 200 million chess positions per second. Following the match, Kasparov remarked that Deep Blue was “as intelligent as your alarm clock.”

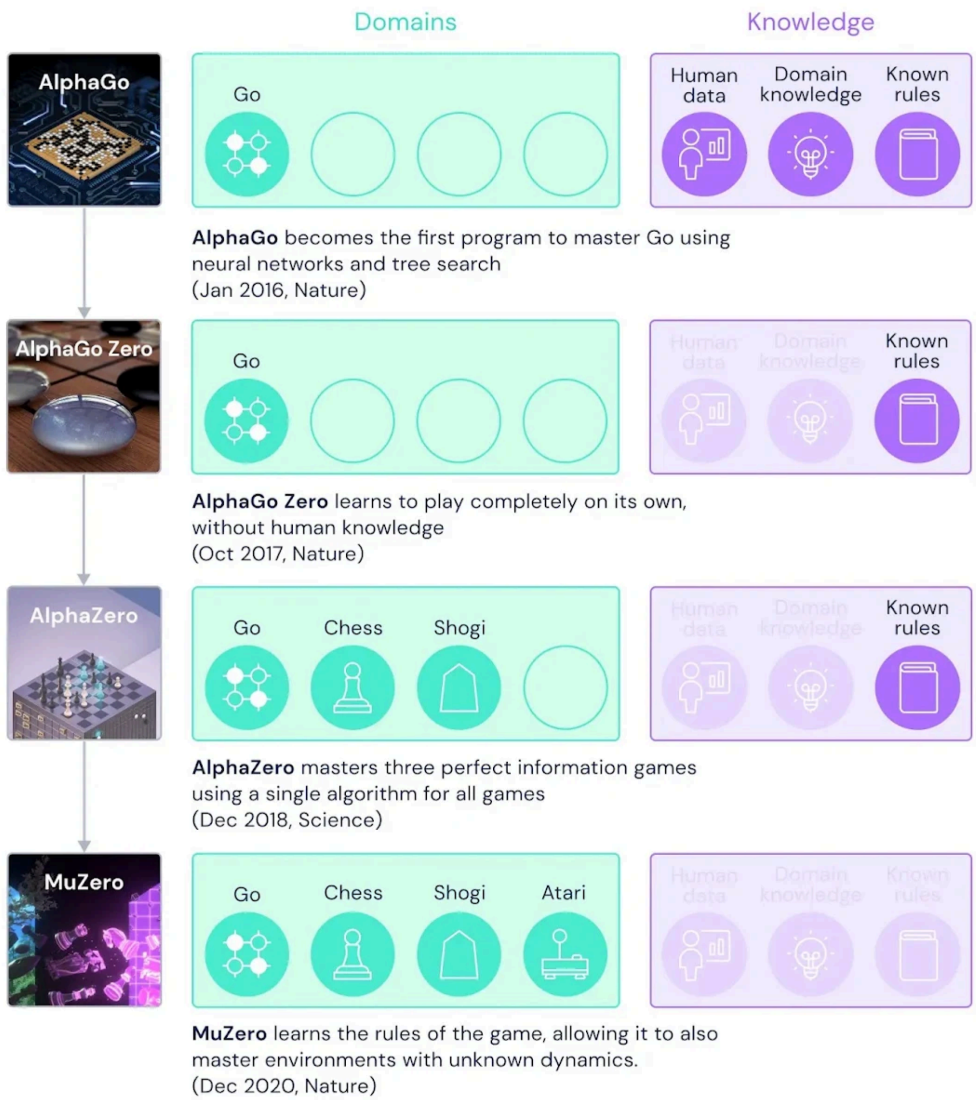
Since then, significant advancements have been made. Unlike Deep Blue, AlphaZero relies on “more interesting approaches than brute-force search, which are perhaps more human-like in the way that they deal with the position.” This prompted Kasparov to express his approval, stating, “I can’t hide my satisfaction that [AlphaZero] plays with a dynamic style reminiscent of my own!”

AlphaGo - The Movie

Yes, **AlphaGo** successfully defeated the **world champion**, Lee **Sedol**. (1**h** 30**m**)

AlphaGo - The Movie | Full award-winning documentary, Google DeepMind, 2020-03-13.

AlphaGo2MuZero



Attribution: MuZero: Mastering Go, chess, shogi and Atari without rules, Google DeepMind, 2020-12-23. Schrittwieser et al. (2020)

David Silver, a principal research scientist at DeepMind and a professor at University College London, is one of the leading researchers on these projects. He earned his PhD at the University of Alberta, where he was supervised by Richard Sutton.

You might also want to watch the following video: AlphaGo Zero: Discovering new knowledge. Posted on YouTube on 2017-10-18.

Search

Applications

- **Pathfinding and Navigation**: Used in robotics and video games to find a **path** from a **starting point** to a **destination**.

- **Puzzle Solving**: Solving puzzles like the **8-puzzle**, **mazes**, or **Sudoku**.
- **Network Analysis**: Analyzing networks and graphs, such as finding **connectivity** or **shortest paths** in **social networks** or **transportation maps**.
- **Game Playing**: Used to evaluate moves in games like **chess** or **Go**, especially when combined with other strategies.

Applications

- **Scheduling**: Planning and scheduling tasks in **manufacturing**, **project management**, or **airline scheduling**.
- **Resource Allocation**: Allocating resources in a network or within an organization where **constraints** must be satisfied.
- **Configuration Problems**: Solving problems where a set of components must be assembled to meet specific requirements, such as configuring a computer system or **designing a circuit**.

Applications

- **Decision Making under Uncertainty**: Used in **real-time strategy games** and **simulations** where decisions need to be evaluated under uncertain conditions.

- **Storrytelling**: LLMs can effectively generate stories when guided by a valid input plan from an **automated planner**. (Simon and Muise 2024)

The applications of search algorithms are both **numerous** and **diverse**.

Search has been an active area of research not only because of its wide range of applications but also due to the potential for improvements in algorithms to significantly reduce program execution time or facilitate the exploration of larger search spaces.

Outline

1. **Deterministic & Heuristic Search**: BFS, DFS, A* for pathfinding and optimization in classical AI.

2. **Population-Based Algorithms**: Focus on structured problems and stochastic search.

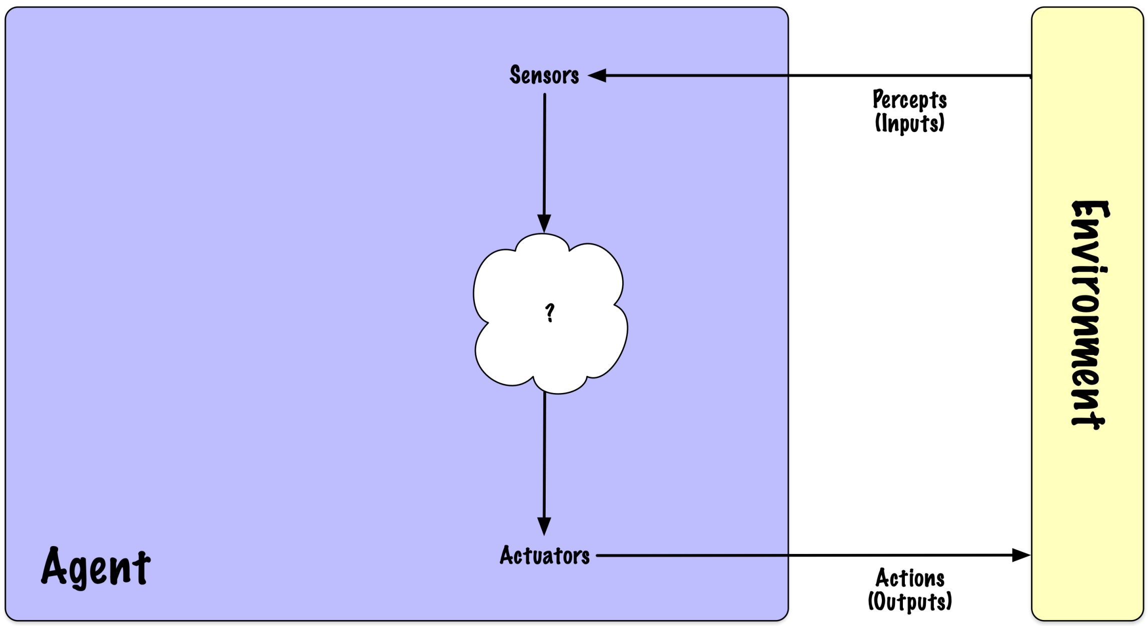
3. **Adversarial Game Algorithms**: Minimax, alpha-beta pruning, MCTS for decision-making in competitive environments.

This lecture and the upcoming ones will thoroughly cover these topics.

Definition

> ** (Russell and Norvig 2020, 63)**
>
> When the correct action to take is not immediately obvious, an agent may need to to **plan ahead**: to consider a **sequence of actions** that form a **path** to a **goal state**. Such an agent is called a **problem-solving agent**, and the computational process it undertakes is called **search**.

Terminology



An **agent** is an entity that performs **actions**. A **rational agent** is one that acts to achieve the **"best" outcome**. Conceptually, an **agent** perceives its environment through **sensors** and interacts with it using **actuators**.

The definition of an agent in **artificial intelligence (AI)** shares some similarities with the **psychological** definition, but there are key distinctions. In AI, an agent is an **autonomous entity** that perceives its **environment** through **sensors** and acts upon it using **actuators** to achieve specific **goals**. Both definitions involve **perception**, **decision-making**, and **action**.

However, while psychological agents are human or biological and involve **complex cognitive** and **emotional processes**, AI agents are computational and operate based on algorithms designed to **maximize certain performance measures** or **achieve predefined objectives**. The focus in AI is more on the technical implementation of these processes, whereas in psychology, the emphasis is on understanding the cognitive and motivational aspects of agency.

The concept of **agentic design** in software engineering and artificial intelligence has experienced a resurgence in popularity.

Environment Characteristics

- **Observability**: Partially observable, or *fully observable*
- **Agent Composition**: *Single* or multiple agents
- **Predictability**: *Deterministic* or non-deterministic
- **State Dependency**: *Stateless* or stateful
- **Temporal Dynamics**: *Static* or dynamic
- **State Representation**: *Discrete* or continuous

In this lecture, environments are assumed to be: *fully observable*, *single agent*, *stateless*, *deterministic*, *static*, and *discrete*.

The characteristics of an environment influence the complexity of problem-solving.

- A **fully observable environment** allows the agent to detect all relevant aspects for decision-making.
- In a **deterministic environment**, the agent can predict the next state based on the current state and its subsequent action.
- **Stateless (Episodic) Environments** involve decisions or actions that are independent of prior actions, with experiences divided into unrelated episodes. An example is a classification problem.
- **Stateful (Sequential) Environments** require that each action's outcome can affect future decisions, as the sequence of actions impacts the state and subsequent choices. An example is a chess game.
- A **dynamic environment** is characterized by changes in context while the agent is deliberating.
- Chess serves as an example of a **discrete environment**, with a finite, though large, number of states. In contrast, an autonomous vehicle operates within a **continuous-state** and **continuous-time environment**.

Problem-Solving Process

Search: The process involves simulating sequences of actions until the agent achieves its goal. A successful sequence is termed a **solution**.

A precise formulation facilitates the development of reusable code.

An environment characterized as stateless, single-agent, fully observable, deterministic, static, and discrete implies that the solution to any problem within this context is a fixed sequence of actions.

- **Stateless**: Each decision is independent of previous actions, meaning the solution does not depend on history.
- **Single Agent**: There is no interaction with other agents that could introduce variability.
- **Fully Observable**: The agent has complete information about the environment, allowing for precise decision-making.
- **Deterministic**: The outcome of actions is predictable, with no randomness affecting the result.
- **Static**: The environment does not change over time, so the conditions remain constant.
- **Discrete**: The environment has a finite number of states and actions, enabling a clear sequence of steps.

I anticipate that you are already familiar with these concepts, and thus, this initial lecture primarily serves as a review.

Search Problem Definition

- A collection of **states**, referred to as the **state space**.
- An **initial state** where the agent begins.
- One or more **goal states** that define successful outcomes.
- A set of **actions** available in a given state s.
- A **transition model** that determines the next state based on the current state and selected action.
- An **action cost function** that specifies the cost of performing action a in state s to reach state s'.

Definitions

- A **path** is defined as a sequence of actions.
- A **solution** is a path that connects the initial state to the goal state.
- An **optimal solution** is the path with the lowest cost among all possible solutions.

We assume that the **path cost** is the sum of the individual action costs, and all costs are positive. The state space can be conceptualized as a **graph**, where the nodes represent the states and the edges correspond to the actions.

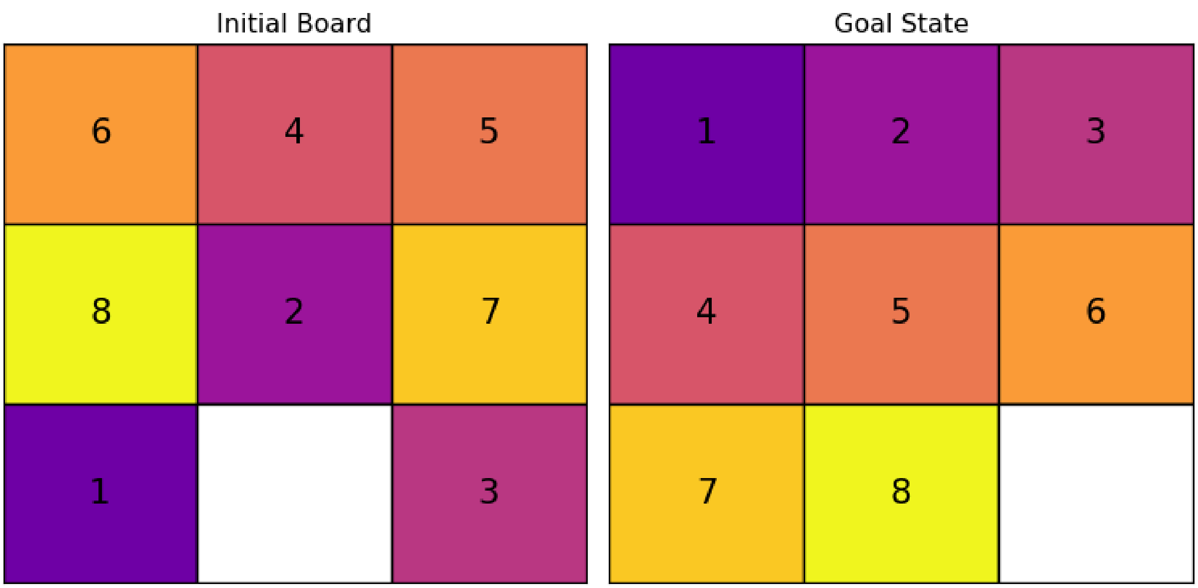
In certain problems, multiple optimal solutions may exist. However, it is typically sufficient to identify and report a single optimal solution. Providing all optimal solutions can significantly increase time and space complexity for some problems.

Example: 8-Puzzle

In [1]:
```
import random
import matplotlib.pyplot as plt
import numpy as np

random.seed(58)

def is_solvable(tiles):
    # Count the inversions in the flattened list of tiles (excluding the bla
    # Assumption: board is square and width is odd.
    inversions = 0
    for i in range(len(tiles)):
        for j in range(i + 1, len(tiles)):
            if tiles[i] != 0 and tiles[j] != 0 and tiles[i] > tiles[j]:
                inversions += 1
    return inversions % 2 == 0

def generate_solvable_board():
    # Generate a random board configuration that is guaranteed to be solvabl
    tiles = list(range(9))
    random.shuffle(tiles)
    while not is_solvable(tiles):
        random.shuffle(tiles)

    return tiles

def plot_board(board, title, num_pos, position):
    ax = plt.subplot(1, num_pos, position)
    ax.set_title(title)
    ax.set_xticks([])
    ax.set_yticks([])

    board = np.array(board).reshape(3, 3).tolist() # Reconfigure the grid to

    # Use a color map to display the numbers
    cmap = plt.cm.plasma
    norm = plt.Normalize(vmin=-1, vmax=8)

    for i in range(3):
        for j in range(3):
            tile_value = board[i][j]
            color = cmap(norm(tile_value))
            ax.add_patch(plt.Rectangle((j, 2 - i), 1, 1, facecolor=color, ec
            if tile_value == 0:
                ax.add_patch(plt.Rectangle((j, 2 - i), 1, 1, facecolor='whit
            else:
                ax.text(j + 0.5, 2 - i + 0.5, str(tile_value),
                        fontsize=16, ha='center', va='center', color='black'

    ax.set_xlim(0, 3)
    ax.set_ylim(0, 3)
```

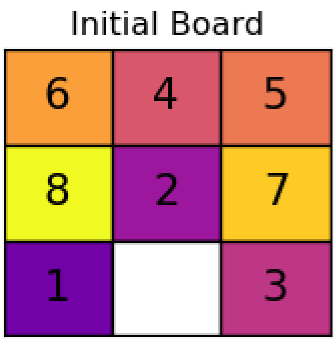

Example: 8-Puzzle

In [2]:
```python
# Generate initial solvable board
initial_board = generate_solvable_board()

# Define goal state
goal_board = [
    [1, 2, 3],
    [4, 5, 6],
    [7, 8, 0]
]

# Plot both boards
plt.figure(figsize=(8, 4))
plot_board(initial_board, "Initial Board", 2, 1)
plot_board(goal_board, "Goal State", 2, 2)
plt.tight_layout()
plt.show()
```



8-Puzzle

- How can the **states** be represented?
- What constitutes the **initial state**?
- What defines the **actions**?
- What would constitute a **path**?
- What characterizes the **goal state**?
- What would constitute a **solution**?
- What should be the **cost of an action**?



- Each state can be represented as a list containing the numbers 0 to 8. Each number corresponds to a tile, and its position in the list reflects its location in the grid, with 0 denoting the blank space.
- The initial state is a permutation of the numbers 0 to 8.
- Actions include **left**, **right**, **up**, and **down**, which involve sliding an adjacent tile into the blank space.
- A path would be a sequence of actions, say **left**, **left**, **up**.
- The transition model maps a given state and action to a new state. Not all actions are feasible from every state; for instance, if the blank space is at the edge of the grid, only certain moves are possible, such as down, up, or left.
- The goal state is achieved when the list is ordered from 1 to 8, followed by 0, indicating that the tiles are arranged correctly. How many goal states are there?
- A solution would be a valid path transforming an initial state into a goal state.
- Each action incurs a cost of 1.

How many possible states are there?

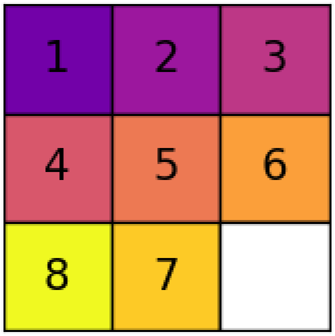
There are 9! = 362,880 states. Brut force is feasible.

How many states are there for the 15-Puzzle?

15! = 1,307,674,368,000 (1.3 trillion)!

Are all the boards solvable?

In [4]:
```

```



Are all the boards solvable?

- A board is solvable if it has the same **inversion parity** as the goal state.
- An **inversion** is a pair of tiles (excluding the blank) that are in the wrong order relative to each other, when reading the board as a one-dimensional list (left-to-right, top-to-bottom).
- When the goal is the ordered sequence from 1 to 8, there are no inversion, and therefore parity is even.
- See Archer (1999) or Slider Puzzle, Princeton

`count_even_odd_parity`

In [5]:

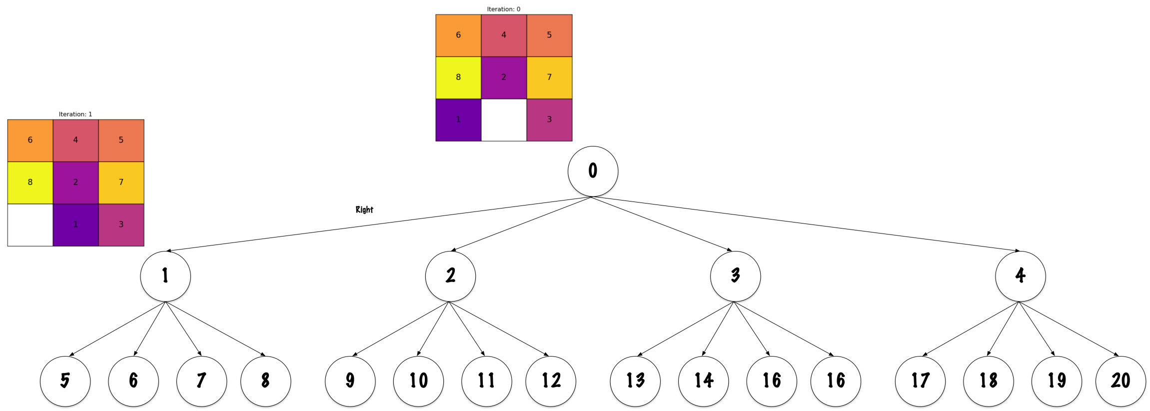
```
from itertools import permutations

def count_even_odd_parity():
    even_count = 0
    odd_count = 0

    for perm in permutations(range(9)):  # 0 represents the blank
        inversions = 0
        for i in range(9):
            for j in range(i + 1, 9):
                if perm[i] != 0 and perm[j] != 0 and perm[i] > perm[j]:
                    inversions += 1
        if inversions % 2 == 0:
            even_count += 1
        else:
            odd_count += 1

    return even_count, odd_count
```

In [6]:

```
even, odd = count_even_odd_parity()

print(f"Even parity: {even}")
print(f"Odd parity: {odd}")
```

```
Even parity: 181440
Odd parity: 181440
```

Search Tree

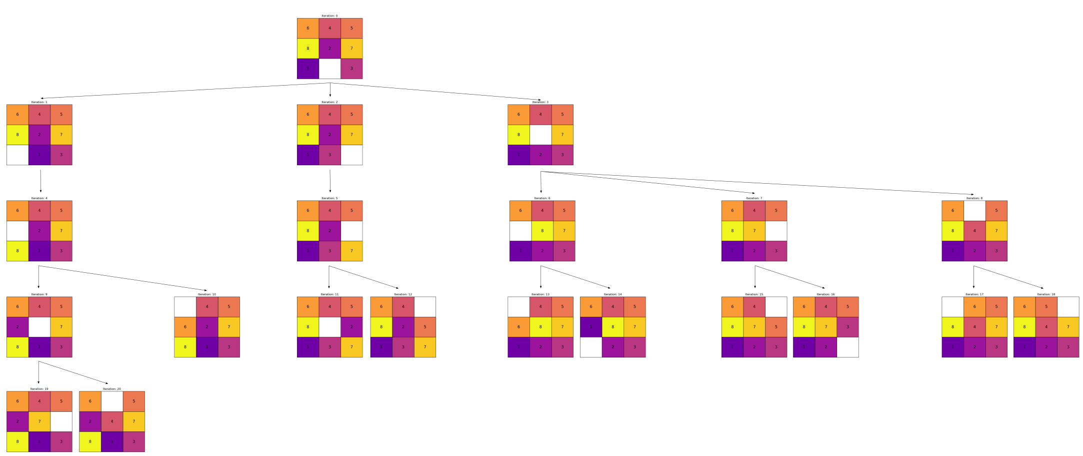


A **search tree** is a *conceptual tree structure* where **nodes** represent *states* in a **state space**, and **edges** represent possible **actions**, facilitating systematic exploration to find a **path** from an **initial state** to a **goal state**.

The search algorithms we examine today construct a search tree, where each node represents a state within the state space and each edge represents an action.

It is important to distinguish between the search tree and the state space, which can be depicted as a graph. The structure of the search tree varies depending on the algorithm employed to address the search problem.

Search Tree



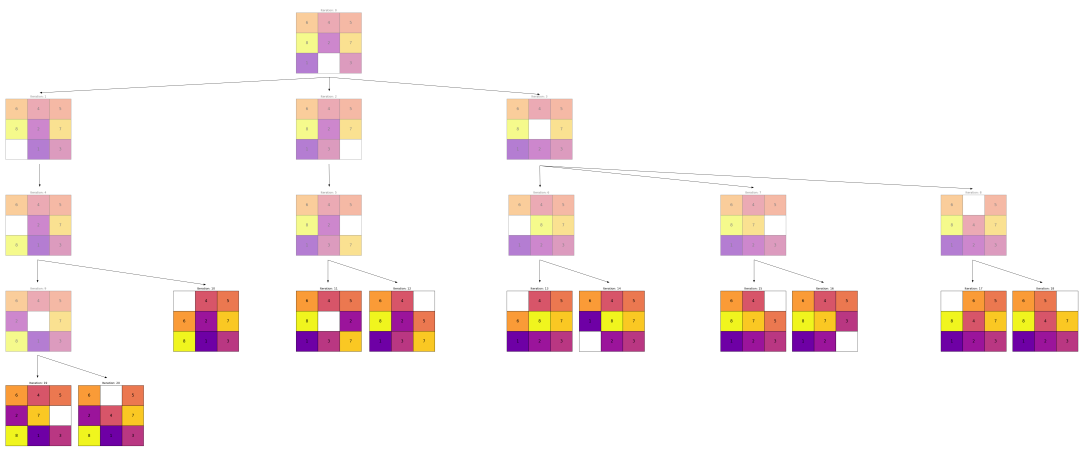
An example of a **search tree** for the **8-Puzzle**. The solution here is incomplete.

Search Tree

- The **root** of the search tree represents the **initial state** of the problem.
- **Expanding** a node involves evaluating **all possible actions** available from that state.
- The **result** of an action is the new state achieved after **applying that action** to the **current state**.
- Similar to other tree structures, each node (except for the **root** and **leaf** nodes) has a **parent** and may have **children**.

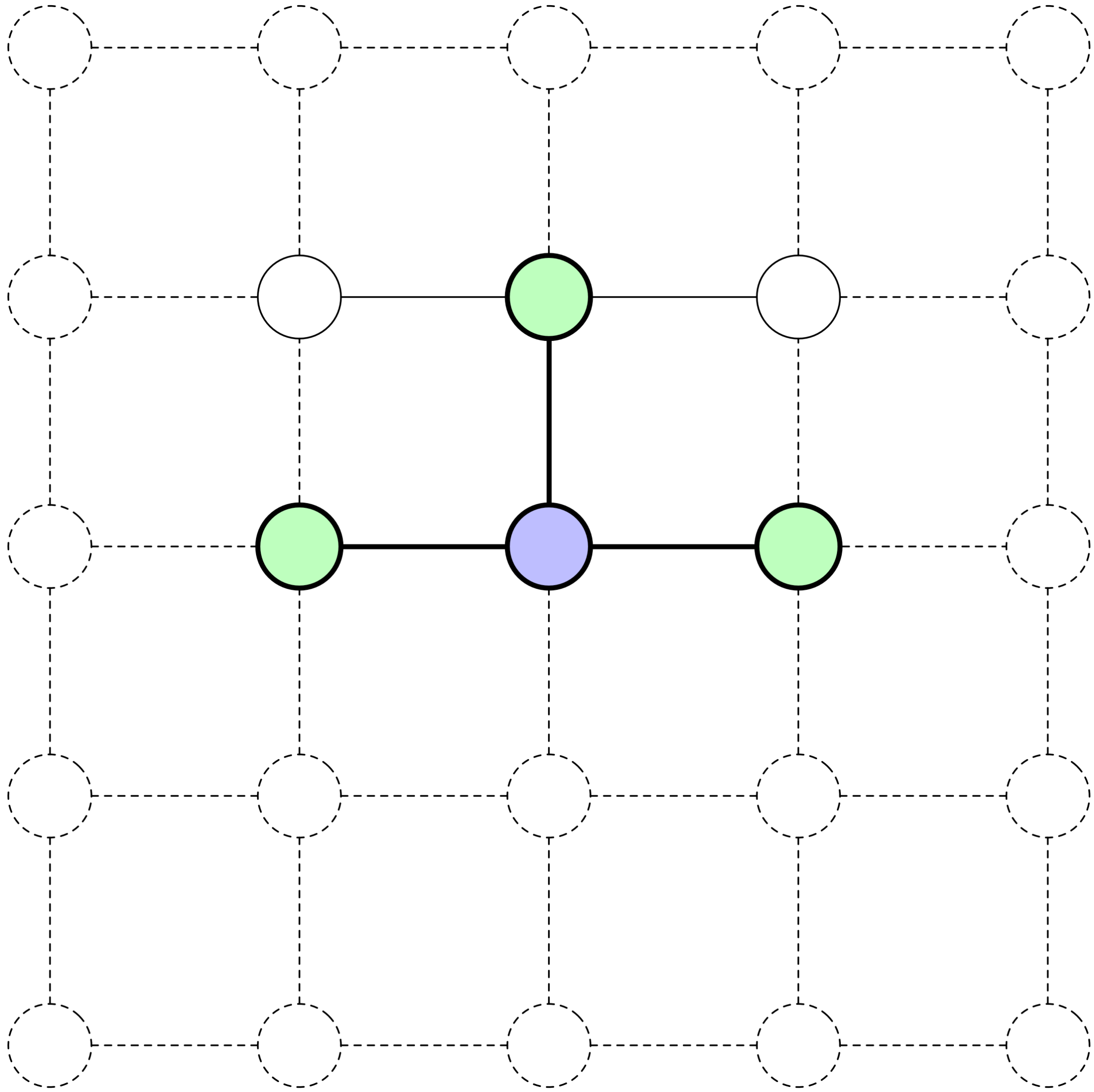
A distinctive feature of a search algorithm is its method for **selecting the next node to expand**.

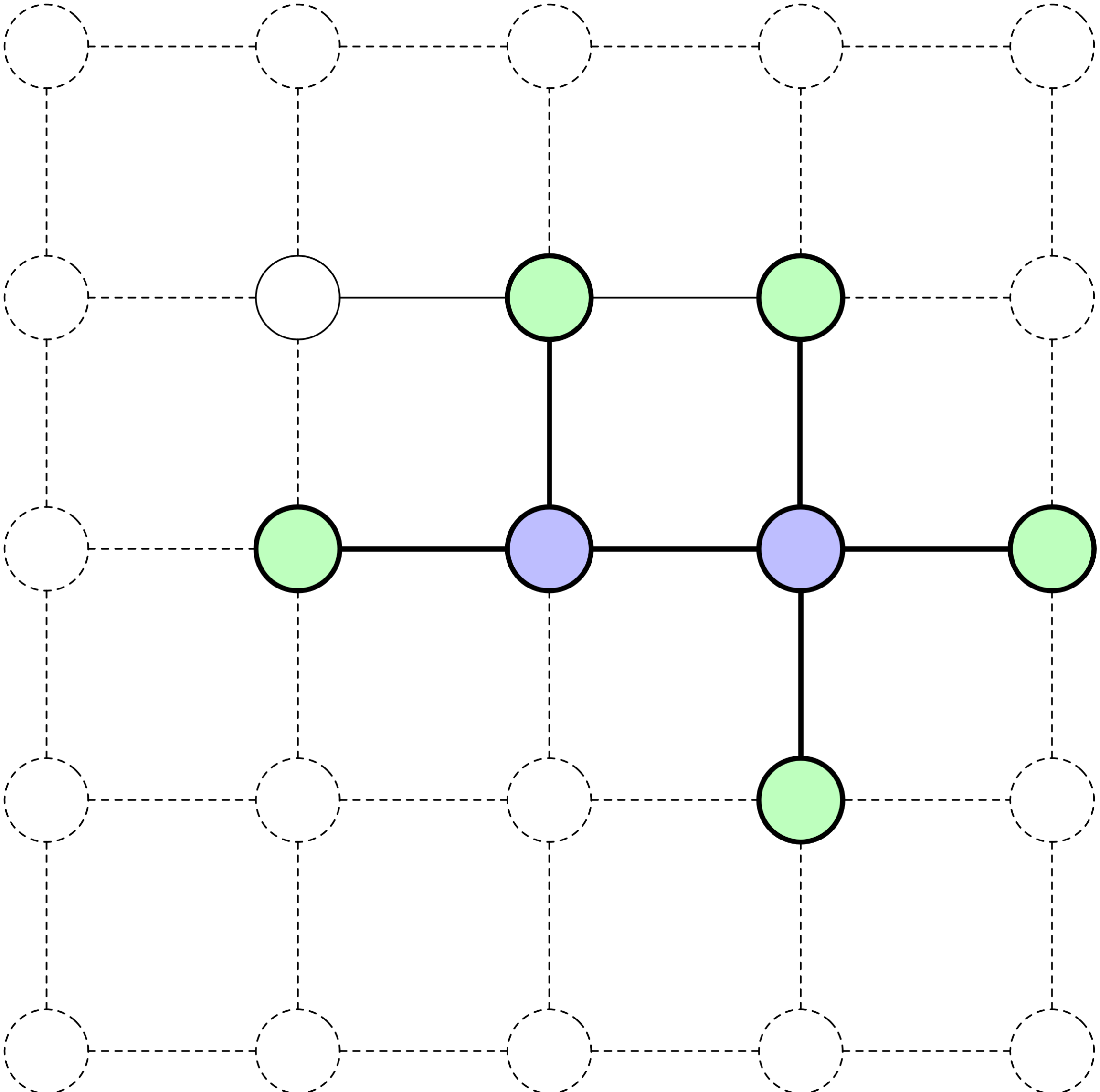
Frontier

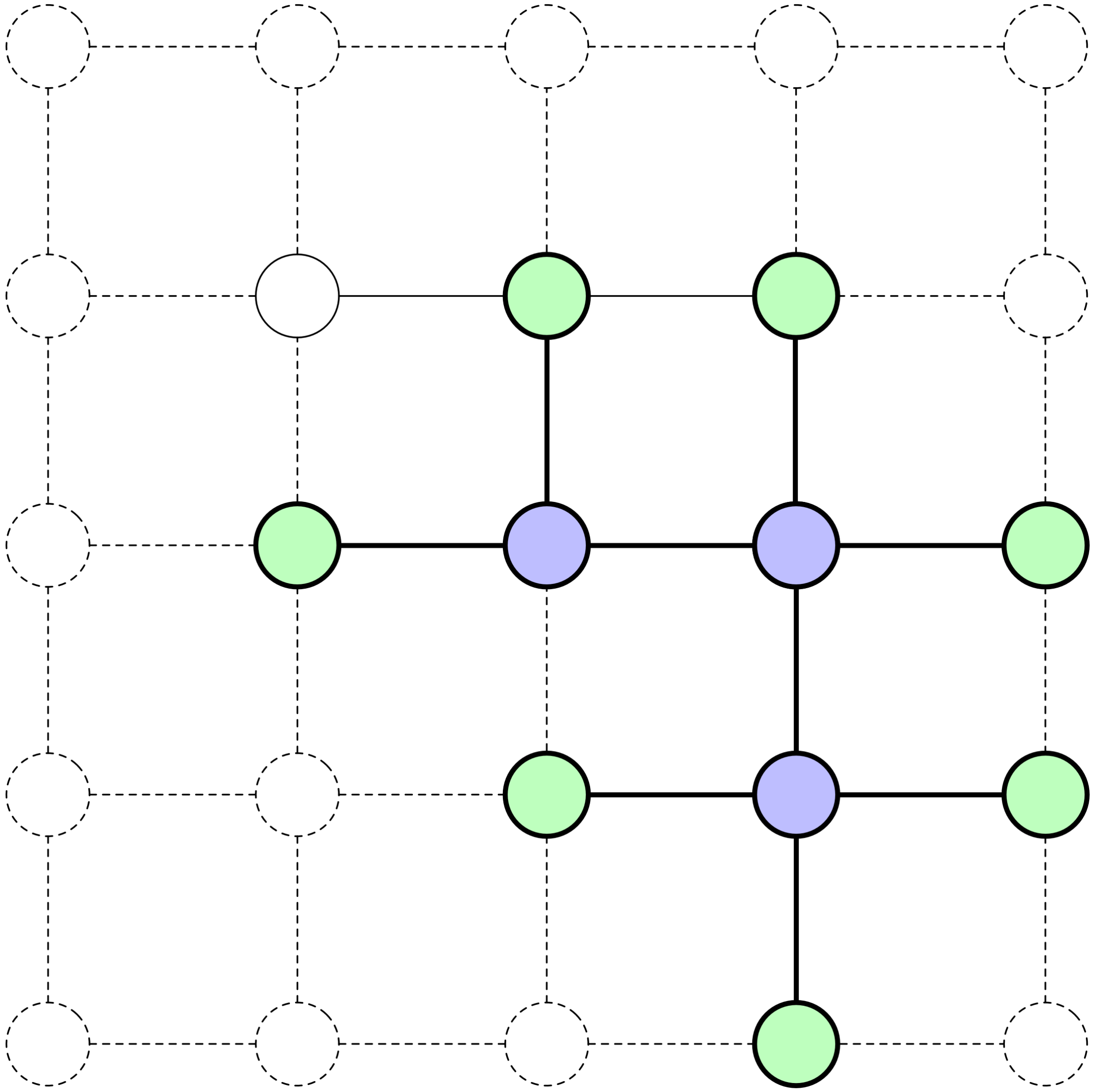


Any state corresponding to a node in the search tree is considered **reached**. **Frontier** nodes are those that have been **reached** but have **not yet been expanded**. Above, there are **10 expanded nodes** and **11 frontier nodes**, resulting in a total of 21 nodes that have been **reached**.

Frontier



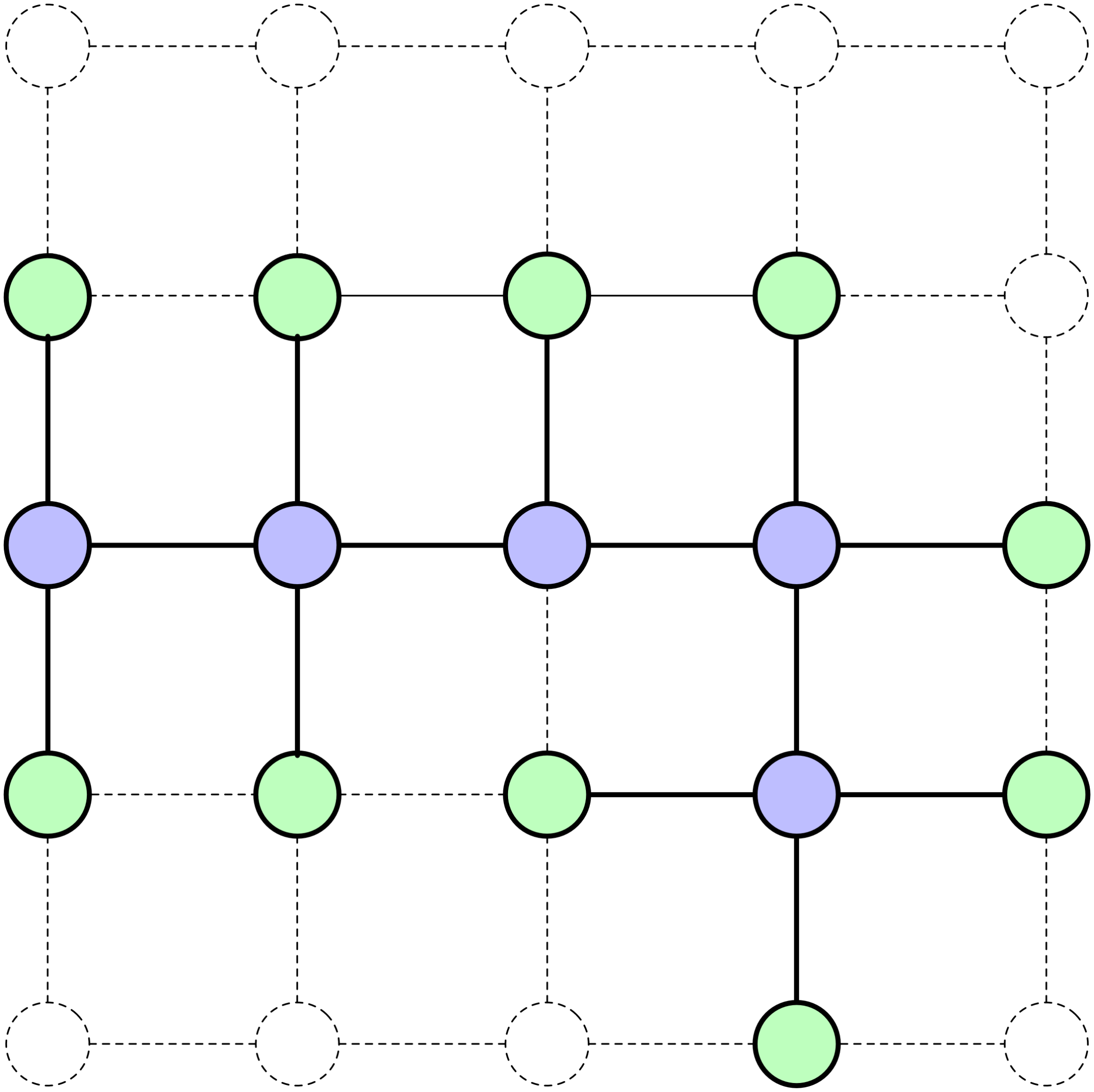


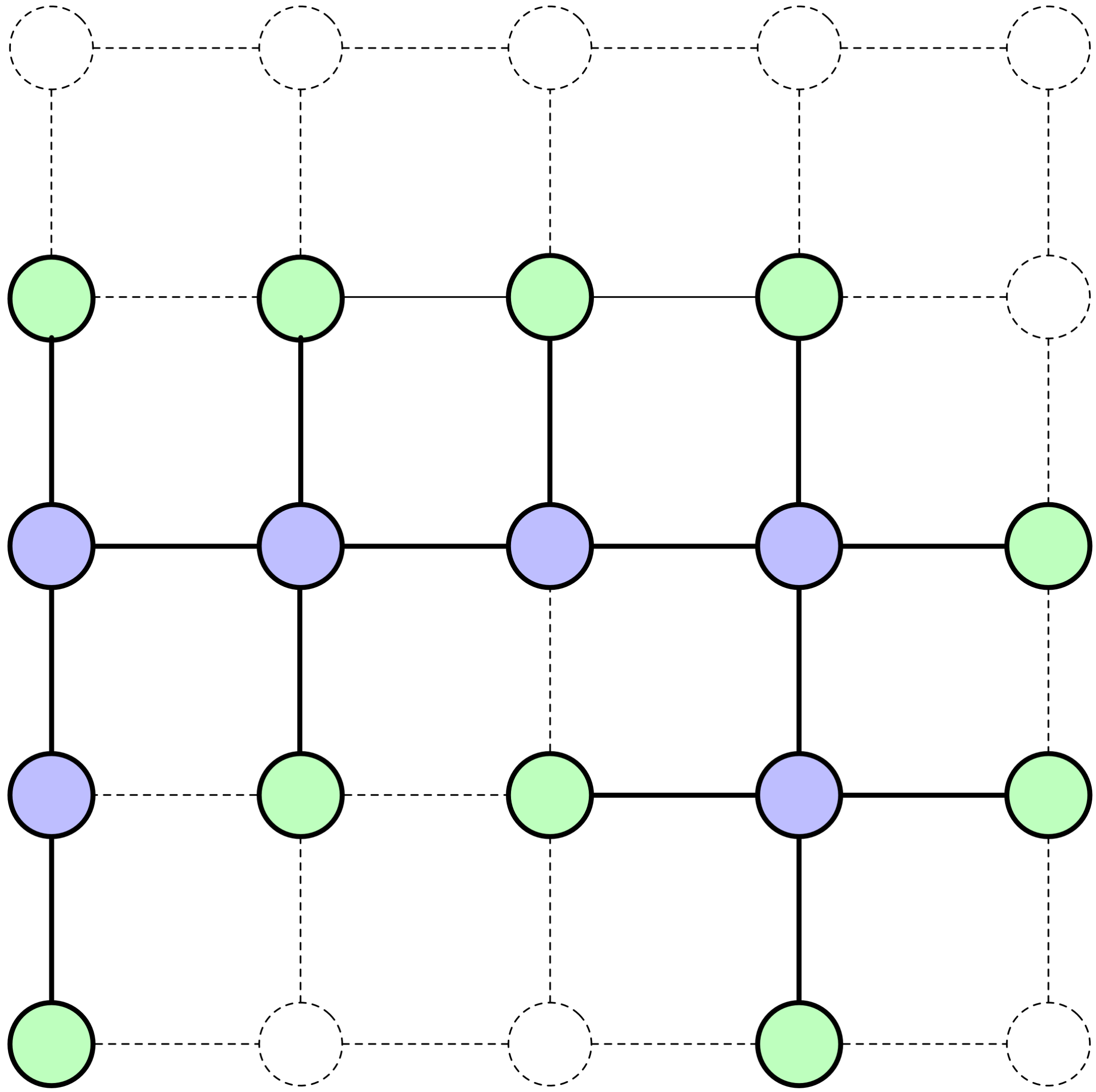


In the 8-Puzzle, four actions are possible: slide **left**, **right**, **up**, or **down**. The search can be visualized on a grid: **purple nodes**: expanded states, **green nodes**: frontier states (reached but not expanded).

The diagrams correspond to the search tree presented on the previous page. For example, the initial state can be expanded using three actions: slide left, right, and up. Node (2, 3) can only be expanded by sliding down, while node (3, 3) can be expanded by sliding left and down.

Frontier





Uninformed search

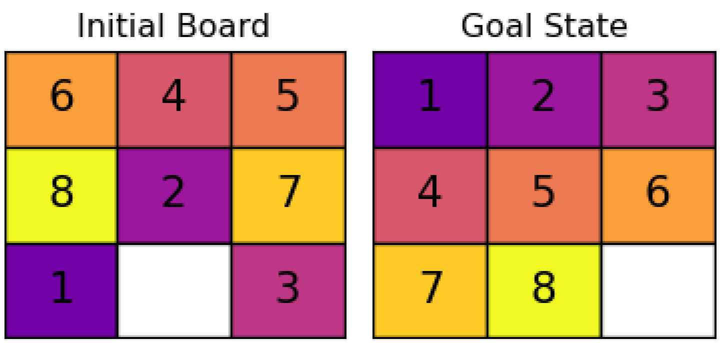
Definition

An **uninformed search** (or blind search) is a search strategy that explores the search space using only the information available in the problem definition, without any domain-specific knowledge, evaluating nodes based solely on their inherent properties rather than estimated costs or heuristics.

State Representation

In [7]:
```
# Plot both boards
initial_state_8 = [6, 4, 5,
                   8, 2, 7,
                   1, 0, 3]
```

```
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

plt.figure(figsize=(4, 2))
plot_board(initial_state_8, "Initial Board", 2, 1)
plot_board(goal_state_8, "Goal State", 2, 2)
plt.tight_layout()
plt.show()
```



In [8]:
```
initial_state_8 = [6, 4, 5,
                   8, 2, 7,
                   1, 0, 3]

goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]
```

The states are represented as **lists** of numbers. `0` represents the **blank tile**.

`is_goal`

In [9]:
```
def is_goal(state, goal_state):
    """Determines if a given state matches the goal state."""
    return state == goal_state
```

Auxilliary method.

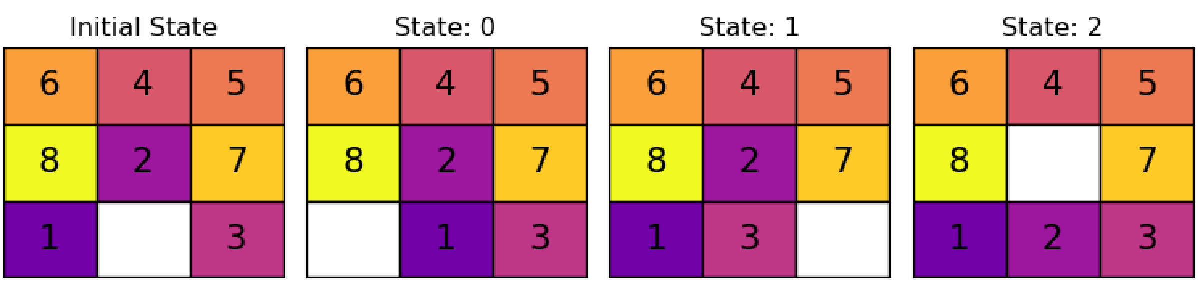
`expand`

In [10]:
```
def expand(state):
    """Generates successor states by moving the blank tile in all possible d
    size = int(len(state) ** 0.5)  # Determine puzzle size (3 for 8-puzzle,
    idx = state.index(0)  # Find the index of the blank tile represented by
    x, y = idx % size, idx // size  # Convert index to (x, y) coordinates
    neighbors = []

    # Define possible moves: Left, Right, Up, Down
    moves = [(-1, 0), (1, 0), (0, -1), (0, 1)]
    for dx, dy in moves:
```

```
        nx, ny = x + dx, y + dy
        # Check if the new position is within the puzzle boundaries
        if 0 <= nx < size and 0 <= ny < size:
            n_idx = ny * size + nx
            new_state = state.copy()
            # Swap the blank tile with the adjacent tile
            new_state[idx], new_state[n_idx] = new_state[n_idx], new_state[i
            neighbors.append(new_state)
    return neighbors
```

`expand`

In [11]:
```
plt.figure(figsize=(8, 2))

solutions = expand(initial_state_8)

plot_board(initial_state_8, "Initial State", 4, 1)

for i, solution in enumerate(solutions):
    plot_board(solution, f"State: {i}", 4, i+2)

plt.tight_layout()
plt.show()
```



In [12]:
```
expand(initial_state_8)
```

```
[[6, 4, 5, 8, 2, 7, 0, 1, 3],
 [6, 4, 5, 8, 2, 7, 1, 3, 0],
 [6, 4, 5, 8, 0, 7, 1, 2, 3]]
```

`is_empty`

In [13]:
```
def is_empty(frontier):
    """Checks if the frontier is empty."""
    return len(frontier) == 0
```

If the frontier becomes empty (no more nodes to be expanded), the problem has no solution.

Are there 8-Puzzle boards that have no solutions?

The solvability of the 8-puzzle depends on the number of **inversions** in the initial state. An inversion is a pair of tiles where a higher-numbered tile precedes a lower-numbered tile when the puzzle is viewed as a sequence (excluding the blank tile).

If the initial state and the goal state do not have the same inversion parity, then the board has no solution.

`print_solution`

In [14]:
```
def print_solution(solution):
    """Prints the sequence of steps from the initial to the goal state."""
    size = int(len(solution[0]) ** 0.5)
    for step, state in enumerate(solution):
        print(f"Step {step}:")
        for i in range(size):
            row = state[i*size:(i+1)*size]
            print(' '.join(str(n) if n != 0 else ' ' for n in row))
        print()
```

Cycles

...

A path that revisits the same states forms a **cycle**.

Allowing cycles would render the resulting **search tree infinite**.

To prevent this, we monitor the states that have been reached, though this incurs a **memory cost**.

Breadth-first search

Breadth-first search

In [15]:
```
from collections import deque
```

Breadth-first search (BFS) employs a **queue** to manage the frontier nodes, which are also known as the open list.

Breadth-first search

In [16]:
```
def bfs(initial_state, goal_state):

    frontier = deque()  # Initialize the queue for BFS
```

```
    frontier.append((initial_state, []))  # Each element is a tuple: (state,

    explored = set()
    explored.add(tuple(initial_state))

    iterations = 0 # simply used to compare algorithms

    while not is_empty(frontier):
        current_state, path = frontier.popleft()

        if is_goal(current_state, goal_state):
            print(f"Number of iterations: {iterations}")
            return path + [current_state]  # Return the successful path

        iterations = iterations + 1

        for neighbor in expand(current_state):
            neighbor_tuple = tuple(neighbor)
            if neighbor_tuple not in explored:
                explored.add(neighbor_tuple)
                frontier.append((neighbor, path + [current_state]))

    return None  # No solution found
```

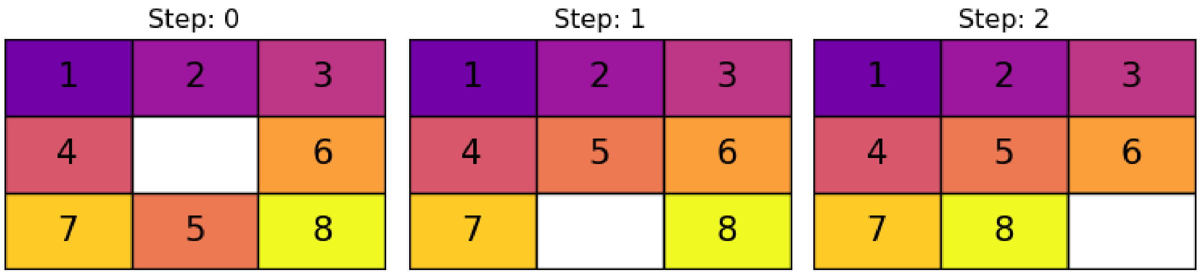
Find the **shortest path** from the initial state to the goal state.

Simple Case

In [17]:

```
plt.figure(figsize=(8, 2))

initial_state_8 = [1, 2, 3,
                   4, 0, 6,
                   7, 5, 8]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

solutions = bfs(initial_state_8, goal_state_8)

for i, solution in enumerate(solutions):
    plot_board(solution, f"Step: {i}", 3, i+1)

plt.tight_layout()
plt.show()
```

```
Number of iterations: 12
```



In [18]:
```
initial_state_8 = [1, 2, 3,
                   4, 0, 6,
                   7, 5, 8]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

bfs(initial_state_8, goal_state_8)
```

```
Number of iterations: 12
[[1, 2, 3, 4, 0, 6, 7, 5, 8],
 [1, 2, 3, 4, 5, 6, 7, 0, 8],
 [1, 2, 3, 4, 5, 6, 7, 8, 0]]
```

Challenging Case

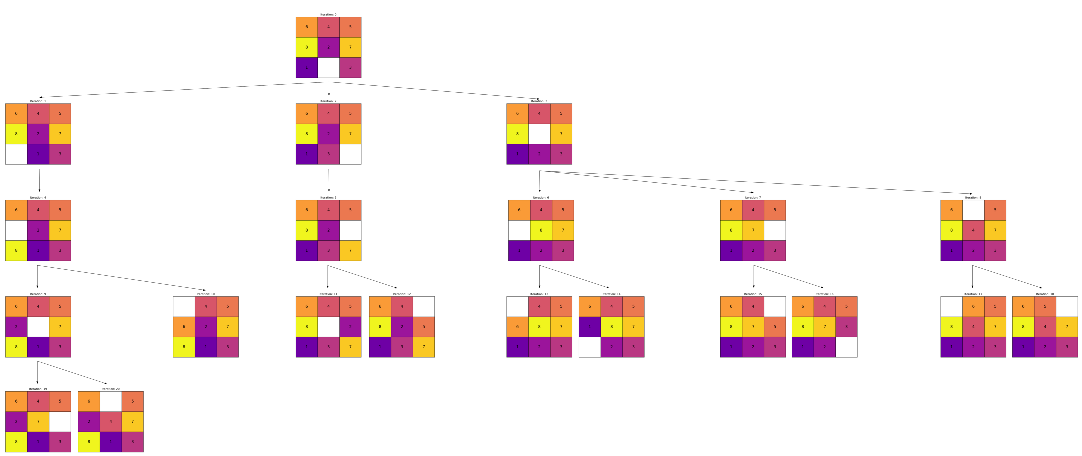
In [19]:
```
initial_state_8 = [6, 4, 5,
                   8, 2, 7,
                   1, 0, 3]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

print("Solving 8-puzzle with BFS...")

solution_8_bfs = bfs(initial_state_8, goal_state_8)

if solution_8_bfs:
    print(f"BFS Solution found in {len(solution_8_bfs) - 1} moves:")
    print_solution(solution_8_bfs)
else:
    print("No solution found for 8-puzzle using BFS.")
```

```
Solving 8-puzzle with BFS...
Number of iterations: 145605
BFS Solution found in 25 moves:
Step 0:
6 4 5
8 2 7
1   3

Step 1:
6 4 5
8 2 7
  1 3

Step 2:
6 4 5
  2 7
8 1 3

Step 3:
6 4 5
2   7
8 1 3

Step 4:
6   5
2 4 7
8 1 3

Step 5:
  6 5
2 4 7
8 1 3

Step 6:
2 6 5
  4 7
8 1 3

Step 7:
2 6 5
4   7
8 1 3

Step 8:
2 6 5
4 1 7
8   3

Step 9:
2 6 5
4 1 7
  8 3

Step 10:
2 6 5
  1 7
```

```
4 8 3

Step 11:
2 6 5
1   7
4 8 3

Step 12:
2 6 5
1 7
4 8 3

Step 13:
2 6 5
1 7 3
4 8

Step 14:
2 6 5
1 7 3
4   8

Step 15:
2 6 5
1   3
4 7 8

Step 16:
2   5
1 6 3
4 7 8

Step 17:
2 5
1 6 3
4 7 8

Step 18:
2 5 3
1 6
4 7 8

Step 19:
2 5 3
1   6
4 7 8

Step 20:
2   3
1 5 6
4 7 8

Step 21:
  2 3
1 5 6
4 7 8
```

```
Step 22:
1 2 3
  5 6
4 7 8

Step 23:
1 2 3
4 5 6
  7 8

Step 24:
1 2 3
4 5 6
7   8

Step 25:
1 2 3
4 5 6
7 8
```

BFS Search Tree



The search tree above illustrates the first 20 iterations of the breadth-first search (BFS) for the specified initial goal.

Depth-First Search

Depth-First Search

In [20]:
```python
def dfs(initial_state, goal_state):

    frontier = [(initial_state, [])]  # Each element is a tuple: (state, pat

    explored = set()
    explored.add(tuple(initial_state))
```

```
    iterations = 0

    while not is_empty(frontier):
        current_state, path = frontier.pop()

        if is_goal(current_state, goal_state):
            print(f"Number of iterations: {iterations}")
            return path + [current_state]  # Return the successful path

        iterations = iterations + 1

        for neighbor in expand(current_state):
            neighbor_tuple = tuple(neighbor)
            if neighbor_tuple not in explored:
                explored.add(neighbor_tuple)
                frontier.append((neighbor, path + [current_state]))

    return None  # No solution found
```

What is the behaviour of **depth-First Search (dfs)**?

Depth-First Search (DFS) consistently expands the deepest node.

When does the deepening process halt?

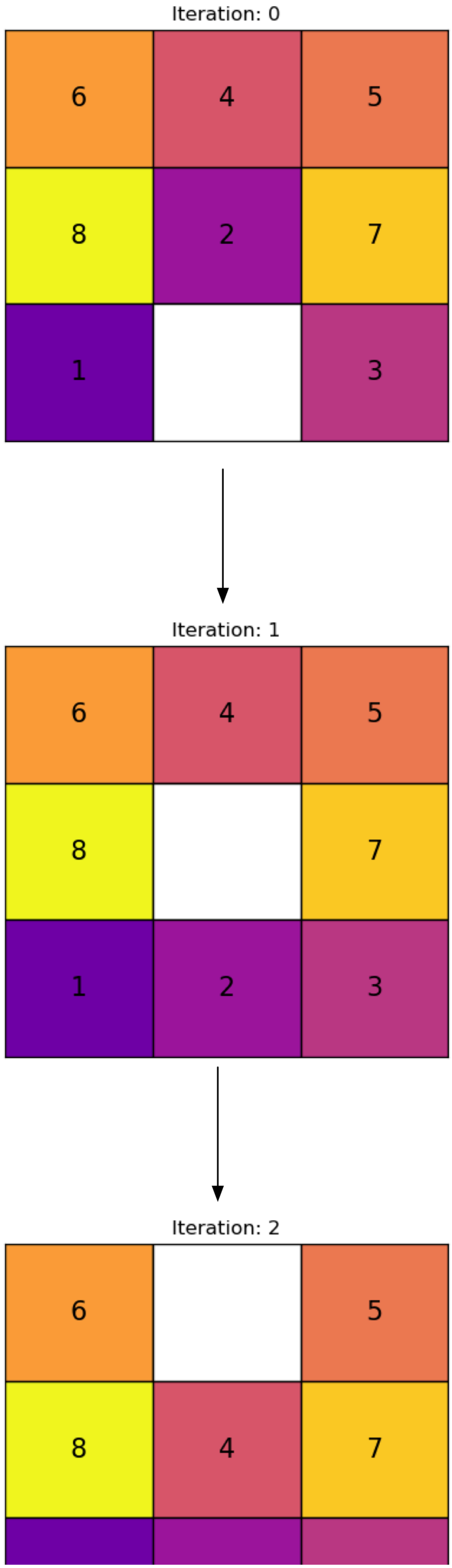
It ceases when all child nodes correspond to states that have already been visited.

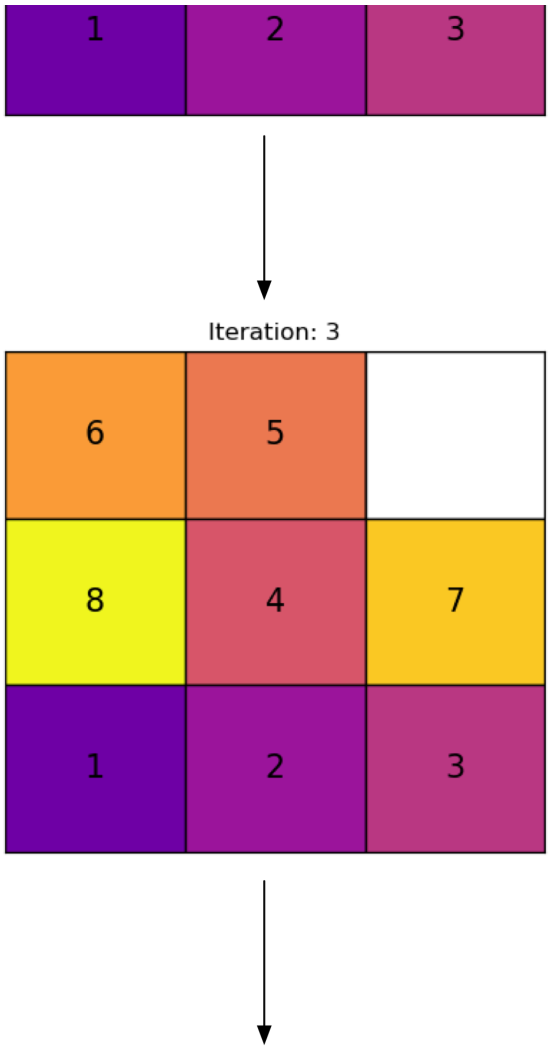
What happens next?

The algorithm backtracks to the most recent frontier node.

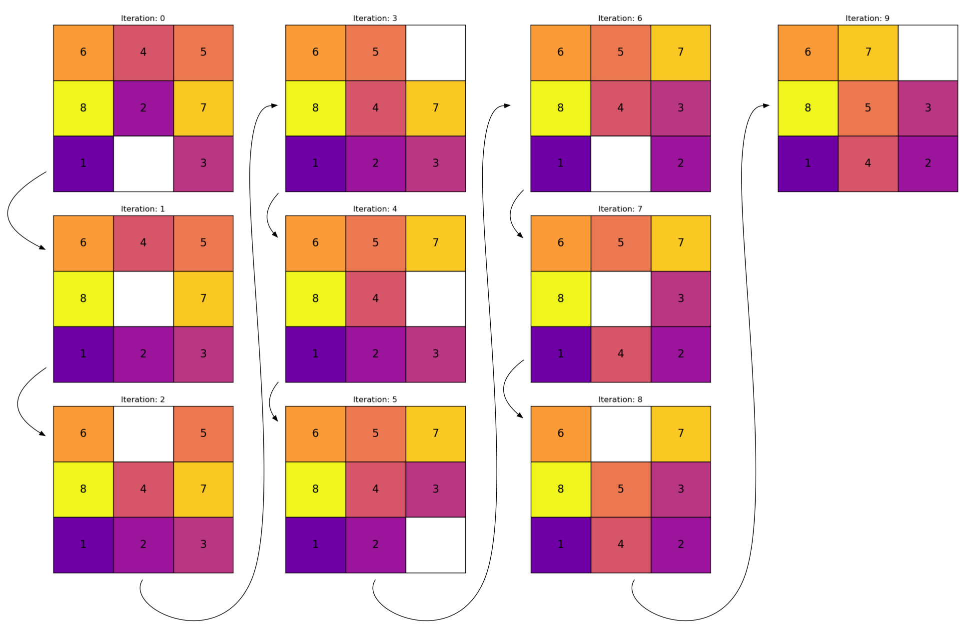
If all children of this node also correspond to previously visited states, the algorithm continues to backtrack further.

DFS Search Tree


Iteration: 0
6 4 5
8 2 7
1 3
Iteration: 1
6 4 5
8 7
1 2 3
Iteration: 2
6 5
8 4 7



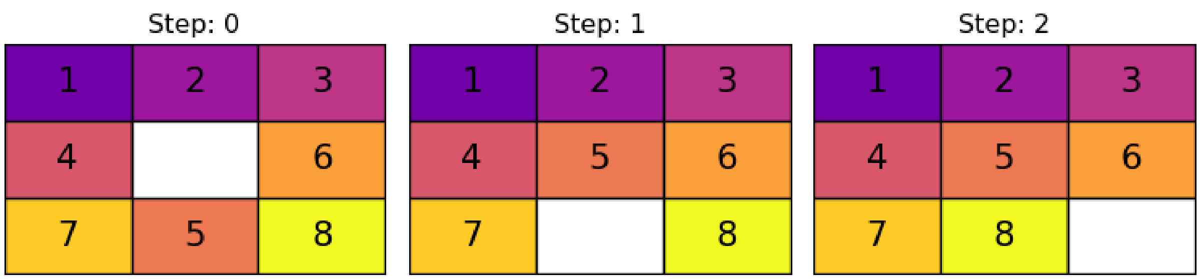
DFS Search Tree



Simple Case

In [21]:
```python
plt.figure(figsize=(8, 2))

initial_state_8 = [1, 2, 3,
                   4, 0, 6,
                   7, 5, 8]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

solutions = dfs(initial_state_8, goal_state_8)

for i, solution in enumerate(solutions):
    plot_board(solution, f"Step: {i}", 3, i+1)

plt.tight_layout()
plt.show()
```

```
Number of iterations: 2
```



In [22]:
```python
initial_state_8 = [1, 2, 3,
                   4, 0, 6,
                   7, 5, 8]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

bfs(initial_state_8, goal_state_8)
```

```
Number of iterations: 12
[[1, 2, 3, 4, 0, 6, 7, 5, 8],
 [1, 2, 3, 4, 5, 6, 7, 0, 8],
 [1, 2, 3, 4, 5, 6, 7, 8, 0]]
```

Challenging Case

In [23]:
```python
initial_state_8 = [6, 4, 5,
                   8, 2, 7,
                   1, 0, 3]
goal_state_8 = [1, 2, 3,
                4, 5, 6,
                7, 8, 0]

print("Solving 8-puzzle with DFS...")

solution_8_bfs = dfs(initial_state_8, goal_state_8)
```

```python
if solution_8_bfs:
    print(f"DFS Solution found in {len(solution_8_bfs) - 1} moves:")
    # print_solution(solution_8_bfs)
else:
    print("No solution found for 8-puzzle using DFS.")
```

```
Solving 8-puzzle with DFS...
Number of iterations: 1187
DFS Solution found in 1157 moves:
```

Remarks

- **Breadth-first search (BFS)** identifies the optimal solution, 25 moves, in 145,605 iterations.

- **Depth-first search (DFS)** discovers a solution involving 1,157 moves in 1,187 iterations.

How can solutions be discovered **more efficiently**?

Will **Depth-First Search (DFS)** invariably yield sub-optimal solutions?

No, if the optimal solution lies along the path traversed by depth-first search (DFS) within the search tree, then DFS will indeed identify the optimal solution.

Is it possible for DFS to discover solutions superior to the optimal solution?

Certainly not; such solutions would either be invalid (involving impossible moves) or indicate an error in your estimation.

Does this imply that depth-first search (DFS) has no practical applications?

When is it appropriate to use DFS?

Breadth-first search (BFS) expands its frontier systematically in all directions, leading to rapid growth in memory requirements.

In contrast, the memory usage of DFS is constrained by the number of moves needed to reach its backtracking points or the path length of the first solution found. In all scenarios, DFS continues expanding the frontier in one direction.

In certain applications where all possible solutions must be explored, the entire search space must be traversed. Using BFS in these cases would be prohibitively expensive in terms of memory. However, DFS can explore the entire space with minimal memory usage.

The programming language Prolog includes a built-in backtracking algorithm that enumerates all possible solutions. Backtracking is a memory-efficient variant of DFS.

Depth-limited and iterative deepening search would be alternative uninformed search algorithms.

Finding solutions more efficiently requires domain knowledge.

Prologue

Summary

- Justification for Studying Search
- Key Terminology and Concepts
- Uninformed Search Algorithms
    - Breadth-First Search (BFS)
    - Depth-First Search (DFS)
- Implementations

- **Justification for Studying Search:**
    - Emphasized the shift from solely focusing on machine learning to incorporating search algorithms.
    - Highlighted the role of search in advanced AI systems like AlphaGo, AlphaZero, and MuZero.
    - Noted that search algorithms are crucial for planning, reasoning, and will be increasingly significant.
- **Historical Timeline of Search Algorithms:**
    - Presented a biased timeline from 1968's A* algorithm to recent developments like MuZero and Agent57.
    - Showed the evolution from heuristic-based search to integrating deep learning with search methods.
- **Applications of Search:**
    - **Pathfinding and Navigation:** Finding optimal paths in robotics and games.
    - **Puzzle Solving:** Solving problems like the 8-puzzle and Sudoku.
    - **Network Analysis:** Analyzing connectivity and shortest paths in networks.
    - **Game Playing:** Evaluating moves in games like chess or Go.
    - **Scheduling and Resource Allocation:** Planning tasks and allocating resources efficiently.
    - **Configuration Problems:** Assembling components to meet specific requirements.

- **Decision Making under Uncertainty:** Making decisions in dynamic and uncertain environments.
- **Storytelling:** Guiding language models with plans from automated planners.

- **Key Terminology and Concepts:**
  - **Agent:** An entity that performs actions to achieve goals.
  - **Environment Characteristics:** Fully observable, single-agent, deterministic, static, and discrete environments were focused on.
  - **Search Problem Definition:**
    - **State Space:** All possible states.
    - **Initial State:** Where the agent starts.
    - **Goal State(s):** Desired outcome(s).
    - **Actions:** Possible moves from a state.
    - **Transition Model:** Rules determining state changes.
    - **Action Cost Function:** Cost associated with actions.
- **Uninformed Search Algorithms:**
  - **Breadth-First Search (BFS):**
    - Explores the search space level by level.
    - Guarantees the shortest path but can be memory-intensive.
    - Implemented using a queue.
  - **Depth-First Search (DFS):**
    - Explores as deep as possible along each branch before backtracking.
    - Less memory usage but may not find the shortest path.
    - Implemented using a stack.
- **Implementing Uninformed Search:**
  - Used the **8-Puzzle** as an example problem.
  - Represented states as lists of numbers, with `0` as the blank tile.
  - Demonstrated BFS and DFS implementations in Python.
  - Showed that BFS found the optimal solution in more iterations, while DFS found a suboptimal solution faster.
- **Limitations of Uninformed Search:**
  - Inefficient for large or complex problems due to exhaustive nature.
  - Lack of domain knowledge leads to unnecessary exploration.

Next lecture

- We will further explore heuristic functions and examine additional search algorithms.

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