Lecture 16

0/1 Knapsack Problem

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Problem

0/1 knapsack problem: Given items with defined **weights** and **values**, the objective is to **maximize total value** by selecting items for a knapsack without surpassing a **fixed capacity**. Each item must be either fully **included (1)** or **excluded (0)**.

Attribution: Generated by DALL-E, via ChatGPT (GPT-4), OpenAI, November 10, 2024.

Utilities

```
import requests

def read_knapsack_data(url):

    """
    Reads and processes knapsack problem data from a given URL.

    Args:
        url (str): The URL pointing to the data file.

    Returns:
        values, weights, capacity
```

```
    Raises:
        Exception: If there is an issue with fetching the data or parsing the content.
    """

    try:
        # Fetch data from the URL
        response = requests.get(url)

        # Raise an error if the request was unsuccessful
        response.raise_for_status()

        # Split the data into lines
        lines = response.text.strip().split('\n')

        # Parse the number of items
        num_items = int(lines[0].strip())

        # Parse the values and weights lists
        values = list(map(int, lines[1].strip().split()))
        weights = list(map(int, lines[2].strip().split()))

        # Parse the capacity
        capacity = int(lines[3].strip())

        # Validate that the number of items matches the length of values and weights
        if len(values) != num_items or len(weights) != num_items:
            raise ValueError("The number of items does not match the length of values or weigh

        # Return the values, weights, and capacity
        return np.array(values), np.array(weights), capacity

    except requests.exceptions.RequestException as e:
        print(f"Error fetching data from URL: {e}")
        raise
    except ValueError as e:
        print(f"Error processing data: {e}")
        raise
    except Exception as e:
        print(f"An unexpected error occurred: {e}")
        raise
```

Greedy Algorithms

> [@Skiena:2008aa, page 192]
>
> Greedy algorithms make the decision of what to do next by selecting the best local option from all available choices without regard to the global structure.

Greedy by weight

A possible greedy strategy involves selecting items in increasing order of weight until the total exceeds the capacity.

```
def greedy_knapsack_weight(weights, values, capacity):

    """
    Greedy algorithm for the 0/1 Knapsack Problem based on weigth.

    Args:
        weights (np.ndarray): Weights of the items.
        values (np.ndarray): Values of the items.
        capacity (int): Capacity of the knapsack.

    Returns:
        tuple: Selected items (binary array), total value, total weight.
    """

    num_items = len(weights)

    # Create a list of items with their values and original indices
    items = list(zip(weights, values, range(num_items)))

    # Sort items by weight in increasing order
    items.sort()

    total_weight = 0
    total_value = 0
    solution = np.zeros(num_items, dtype=int)
```

```
    # Select items based on the sorted order
    for w, v, idx in items:
        if total_weight + w <= capacity:
            solution[idx] = 1
            total_weight += w
            total_value += v
        else:
            continue  # Skip items that would exceed the capacity

    return solution, total_value, total_weight
```

Greedy by value

Another greedy strategy involves selecting items in descending order of value until the total exceeds the capacity.

```
import numpy as np

def greedy_knapsack_value(weights, values, capacity):

    """
    Greedy algorithm for the 0/1 Knapsack Problem based on value.

    Args:
        weights (np.ndarray): Weights of the items.
        values (np.ndarray): Values of the items.
        capacity (int): Capacity of the knapsack.

    Returns:
        tuple: Selected items (binary array), total value, total weight.
    """

    num_items = len(weights)

    # Create a list of items with their values and original indices
    items = list(zip(values, weights, range(num_items)))

    # Sort items by value in decreasing order
    items.sort(reverse=True)
```

```
    total_weight = 0
    total_value = 0
    solution = np.zeros(num_items, dtype=int)

    # Select items based on the sorted order
    for v, w, idx in items:
        if total_weight + w <= capacity:
            solution[idx] = 1
            total_weight += w
            total_value += v
        else:
            continue  # Skip items that would exceed the capacity

    return solution, total_value, total_weight
```

Greedy by ratio

Based on value-to-weight ratio.

```
def greedy_knapsack_ratio(weights, values, capacity):

    """
    Greedy algorithm for the 0/1 Knapsack Problem based on value-to-weight ratio.

    Args:
        weights (np.ndarray): Weights of the items.
        values (np.ndarray): Values of the items.
        capacity (int): Capacity of the knapsack.

    Returns:
        tuple: Selected items (binary array), total value, total weight.
    """

    num_items = len(weights)

    # Calculate value-to-weight ratio for each item
    ratio = values / weights

    # Create a list of items with their ratios and original indices
```

```python
    items = list(zip(ratio, values, weights, range(num_items)))

    # Sort items by ratio in decreasing order
    items.sort(reverse=True)

    total_weight = 0
    total_value = 0
    solution = np.zeros(num_items, dtype=int)

    # Select items based on the sorted order
    for r, v, w, idx in items:
        if total_weight + w <= capacity:
            solution[idx] = 1
            total_weight += w
            total_value += v
        else:
            continue  # Skip items that would exceed the capacity

    return solution, total_value, total_weight
```

Genetic Algorithm

See lecture notes for details.

```python
import random

def initialize_population(pop_size, num_items):
    """
    Initialize the population with random binary strings.

    Args:
        pop_size (int): Number of individuals in the population.
        num_items (int): Number of items in the knapsack problem.

    Returns:
        np.ndarray: Initialized population.
    """
    return np.random.randint(2, size=(pop_size, num_items))
```

```python
def evaluate_fitness(population, weights, values, capacity, penalty_factor=10):
    """
    Evaluate the fitness of each individual in the population.

    Args:
        population (np.ndarray): Current population.
        weights (np.ndarray): Weights of the items.
        values (np.ndarray): Values of the items.
        capacity (int): Capacity of the knapsack.
        penalty_factor (float): Penalty factor for exceeding capacity.

    Returns:
        np.ndarray: Fitness values for the population.
    """
    total_weights = np.dot(population, weights)
    total_values = np.dot(population, values)
    penalties = penalty_factor * np.maximum(0, total_weights - capacity)
    fitness = total_values - penalties
    return fitness
```

```python
def tournament_selection(population, fitness, tournament_size):
    """
    Select individuals from the population using tournament selection.

    Args:
        population (np.ndarray): Current population.
        fitness (np.ndarray): Fitness values of the population.
        tournament_size (int): Number of individuals in each tournament.

    Returns:
        np.ndarray: Selected parents.
    """
    pop_size = population.shape[0]
    selected_indices = []
    for _ in range(pop_size):
        participants = np.random.choice(pop_size, tournament_size, replace=False)
        best = participants[np.argmax(fitness[participants])]
        selected_indices.append(best)
    return population[selected_indices]
```

```python
def roulette_selection(population, fitness):
    """
    Select individuals from the population using roulette wheel selection.

    Args:
        population (np.ndarray): Current population.
        fitness (np.ndarray): Fitness values of the population.

    Returns:
        np.ndarray: Selected parents.
    """
    # Adjust fitness to be non-negative
    min_fitness = np.min(fitness)
    adjusted_fitness = fitness - min_fitness + 1e-6  # small epsilon to avoid zero division
    total_fitness = np.sum(adjusted_fitness)
    probabilities = adjusted_fitness / total_fitness
    pop_size = population.shape[0]
    selected_indices = np.random.choice(pop_size, size=pop_size, p=probabilities)
    return population[selected_indices]
```

```python
def single_point_crossover(parents, crossover_rate):
    """
    Perform single-point crossover on the parents.

    Args:
        parents (np.ndarray): Selected parents.
        crossover_rate (float): Probability of crossover.

    Returns:
        np.ndarray: Offspring after crossover.
    """
    num_parents, num_genes = parents.shape
    np.random.shuffle(parents)
    offspring = []
    for i in range(0, num_parents, 2):
        parent1 = parents[i]
        parent2 = parents[i+1 if i+1 < num_parents else 0]
        child1 = parent1.copy()
        child2 = parent2.copy()
        if np.random.rand() < crossover_rate:
```

```python
            point = np.random.randint(1, num_genes)  # Crossover point
            child1[:point], child2[:point] = parent2[:point], parent1[:point]
        offspring.append(child1)
        offspring.append(child2)
    return np.array(offspring)
```

```python
def uniform_crossover(parents, crossover_rate):
    """
    Perform uniform crossover on the parents.

    Args:
        parents (np.ndarray): Selected parents.
        crossover_rate (float): Probability of crossover.

    Returns:
        np.ndarray: Offspring after crossover.
    """
    num_parents, num_genes = parents.shape
    np.random.shuffle(parents)
    offspring = []
    for i in range(0, num_parents, 2):
        parent1 = parents[i]
        parent2 = parents[i+1 if i+1 < num_parents else 0]
        child1 = parent1.copy()
        child2 = parent2.copy()
        if np.random.rand() < crossover_rate:
            mask = np.random.randint(0, 2, size=num_genes).astype(bool)
            child1[mask], child2[mask] = parent2[mask], parent1[mask]
        offspring.append(child1)
        offspring.append(child2)
    return np.array(offspring)
```

```python
def mutation(offspring, mutation_rate):
    """
    Apply bit-flip mutation to the offspring.

    Args:
        offspring (np.ndarray): Offspring after crossover.
        mutation_rate (float): Probability of mutation for each bit.

```

```
    Returns:
        np.ndarray: Offspring after mutation.
    """
    num_offspring, num_genes = offspring.shape
    mutation_matrix = np.random.rand(num_offspring, num_genes) < mutation_rate
    offspring[mutation_matrix] = 1 - offspring[mutation_matrix]
    return offspring
```

```
def elitism(population, fitness, elite_size):
    """
    Preserve the top-performing individuals in the population.

    Args:
        population (np.ndarray): Current population.
        fitness (np.ndarray): Fitness values of the population.
        elite_size (int): Number of top individuals to preserve.

    Returns:
        np.ndarray: Elite individuals.
    """
    elite_indices = np.argsort(fitness)[-elite_size:]  # Get indices of top individuals
    elites = population[elite_indices]
    return elites
```

```
def genetic_algorithm(weights, values, capacity, pop_size=100, num_generations=200, crossover_r
                      mutation_rate=0.05, elite_percent=0.02, selection_type='tournament', tou
                      crossover_type='single_point'):
    """
    Main function to run the genetic algorithm for the 0/1 knapsack problem.

    Args:
        weights (np.ndarray): Weights of the items.
        values (np.ndarray): Values of the items.
        capacity (int): Capacity of the knapsack.
        pop_size (int): Population size.
        num_generations (int): Number of generations.
        crossover_rate (float): Probability of crossover.
        mutation_rate (float): Probability of mutation.
        elite_percent (float): Percentage of elites to preserve.
        selection_type (str): 'tournament' or 'roulette'.
```

```
        tournament_size (int): Number of individuals in tournament selection.
        crossover_type (str): 'single_point' or 'uniform'.

    Returns:
        tuple: Best solution, best value, and best weight found.
    """

    num_items = len(weights)
    elite_size = max(1, int(pop_size * elite_percent))
    population = initialize_population(pop_size, num_items)

    average_fitness_history = []
    best_fitness_history = []

    for generation in range(num_generations):
        fitness = evaluate_fitness(population, weights, values, capacity)

        # Track average and best fitness
        average_fitness = np.mean(fitness)
        best_fitness = np.max(fitness)
        average_fitness_history.append(average_fitness)
        best_fitness_history.append(best_fitness)

        # Elitism
        elites = elitism(population, fitness, elite_size)

        # Selection
        if selection_type == 'tournament':
            parents = tournament_selection(population, fitness, tournament_size)
        elif selection_type == 'roulette':
            parents = roulette_selection(population, fitness)
        else:
            raise ValueError("Invalid selection type")

        # Crossover
        if crossover_type == 'single_point':
            offspring = single_point_crossover(parents, crossover_rate)
        elif crossover_type == 'uniform':
            offspring = uniform_crossover(parents, crossover_rate)
        else:
```

```
            raise ValueError("Invalid crossover type")

        # Mutation
        offspring = mutation(offspring, mutation_rate)

        # Create new population
        population = np.vstack((elites, offspring))

        # Ensure population size
        if population.shape[0] > pop_size:
            population = population[:pop_size]
        elif population.shape[0] < pop_size:
            # Add random individuals to fill population
            num_new_individuals = pop_size - population.shape[0]
            new_individuals = initialize_population(num_new_individuals, num_items)
            population = np.vstack((population, new_individuals))

    # After all generations, return the best solution
    fitness = evaluate_fitness(population, weights, values, capacity)
    best_index = np.argmax(fitness)
    best_solution = population[best_index]
    best_value = np.dot(best_solution, values)
    best_weight = np.dot(best_solution, weights)

    return best_solution, best_value, best_weight, average_fitness_history, best_fitness_histo
```

```
def genetic_algorithm_do_n(weights, values, capacity, pop_size=100, num_generations=200, crosso
      mutation_rate=0.05, elite_percent=0.02, selection_type='tournament', tournament_size=3, c

    best_solution = None
    best_value = -1
    best_weight = -1

    best_averages = []
    best_bests = []

    for i in range(repeats):

        solution, value, weight, average_history, best_history = genetic_algorithm(weights, val
            mutation_rate, elite_percent, selection_type, tournament_size, crossover_type)
```

```
        if value > best_value and weight <= capacity:
            best_solution = solution
            best_value = value
            best_weight = weight
            best_averages = average_history
            best_bests = best_history

    return best_solution, best_value, best_weight, best_averages, best_bests
```

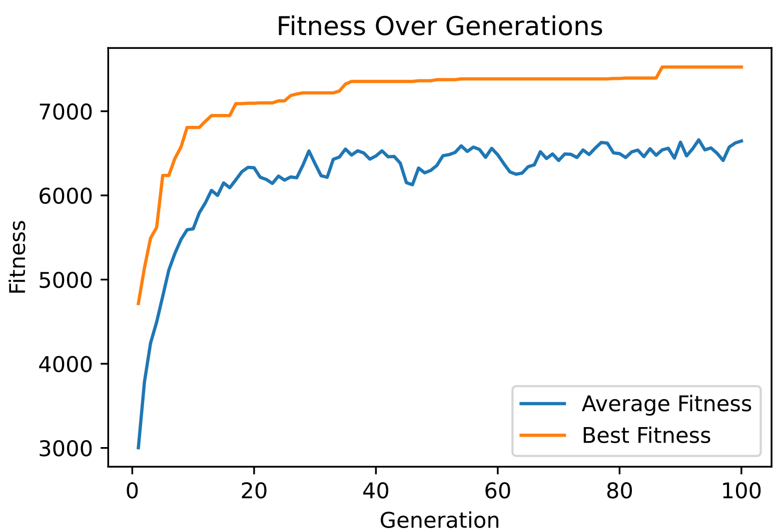
Tests

Testing our genetic algorithm on data from Google OR-Tools.

```
import matplotlib.pyplot as plt

def test_genetic_algorithm():
    # Sample data

    values = np.array([
        360, 83, 59, 130, 431, 67, 230, 52, 93, 125, 670, 892, 600, 38, 48, 147,
        78, 256, 63, 17, 120, 164, 432, 35, 92, 110, 22, 42, 50, 323, 514, 28,
        87, 73, 78, 15, 26, 78, 210, 36, 85, 189, 274, 43, 33, 10, 19, 389, 276,
        312])

    weights = np.array([
        7, 0, 30, 22, 80, 94, 11, 81, 70, 64, 59, 18, 0, 36, 3, 8, 15, 42, 9, 0,
        42, 47, 52, 32, 26, 48, 55, 6, 29, 84, 2, 4, 18, 56, 7, 29, 93, 44, 71,
        3, 86, 66, 31, 65, 0, 79, 20, 65, 52, 13])

    capacity = 850

    # Run genetic algorithm
    best_solution, best_value, best_weight, avg_fitness, best_fitness = genetic_algorithm_do_n
        weights, values, capacity, pop_size=50, num_generations=100,
        crossover_rate=0.8, mutation_rate=0.05, elite_percent=0.02,
        selection_type='tournament', tournament_size=3)

    print("Best Solution:", best_solution)
    print("Best Value:", best_value)
```

```
    print("Best Weight:", best_weight)

    # Plot the fitness over generations
    generations = range(1, len(avg_fitness) + 1)
    plt.plot(generations, avg_fitness, label='Average Fitness')
    plt.plot(generations, best_fitness, label='Best Fitness')
    plt.xlabel('Generation')
    plt.ylabel('Fitness')
    plt.title('Fitness Over Generations')
    plt.legend()
    plt.show()

test_genetic_algorithm()
```

```
Best Solution: [1 1 0 1 1 0 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 0 1 0 0
 0 1 1 0 0 1 0 1 0 0 1 1 1]
Best Value: 7525
Best Weight: 845
```



Testing all the algorithms on data from pages.mtu.edu/~kreher/cages/data/knapsack/.

```
import pandas as pd
```

```
BASE_URL = 'https://pages.mtu.edu/~kreher/cages/data/knapsack/'

datasets = [
    'ks_8a.dat','ks_8b.dat','ks_8c.dat','ks_8d.dat','ks_8e.dat','ks_12a.dat',
    'ks_12b.dat','ks_12c.dat','ks_12d.dat','ks_12e.dat','ks_16a.dat','ks_16b.dat',
    'ks_16c.dat','ks_16d.dat','ks_16e.dat','ks_20a.dat','ks_20b.dat','ks_20c.dat',
    'ks_20d.dat','ks_20e.dat','ks_24a.dat','ks_24b.dat','ks_24c.dat','ks_24d.dat',
    'ks_24e.dat'
]

columns = [
    'file_path', 'capacity',
    'gw_value', 'gw_weight',
    'gv_value', 'gw_weight',
    'gr_value', 'gr_weight',
    'ga_value', 'ga_weight'
]

df = pd.DataFrame(columns=columns)

for idx, file_path in enumerate(datasets):

  values, weights, capacity = read_knapsack_data(BASE_URL + file_path)

  solution, total_value, total_weight = greedy_knapsack_weight(weights, values, capacity)

  gw_value = total_value
  gw_weight = total_weight

  solution, total_value, total_weight = greedy_knapsack_value(weights, values, capacity)

  gv_value = total_value
  gv_weight = total_weight

  solution, total_value, total_weight = greedy_knapsack_ratio(weights, values, capacity)

  gr_value = total_value
  gr_weight = total_weight

  solution, total_value, total_weight, avg_fitness, best_fitness = genetic_algorithm_do_n(
```

```python
        weights, values, capacity, pop_size=50, num_generations=100,
        crossover_rate=0.8, mutation_rate=0.05, elite_percent=0.02,
        selection_type='tournament', tournament_size=3, crossover_type='single_point')

  ga_value = total_value
  ga_weight = total_weight

  df.loc[idx] = [
    file_path, capacity,
    gw_value, gw_weight,
    gv_value, gw_weight,
    gr_value, gr_weight,
    ga_value, ga_weight
  ]

df.to_csv("knapsack.csv", index=False)

print(df)
```

```
     file_path  capacity  gw_value  gw_weight  gv_value  gw_weight  gr_value  \
0     ks_8a.dat   1863633    874414    1803989    925369    1803989    925369
1     ks_8b.dat   1822718    724029    1421763    836649    1421763    724029
2     ks_8c.dat   1609419    771637    1609296    756847    1609296    713452
3     ks_8d.dat   2112292    749458    1558340   1006793    1558340    881823
4     ks_8e.dat   2493250   1224805    2386238   1300939    2386238   1300939
5    ks_12a.dat   2805213   1180238    2323972   1409053    2323972   1381444
6    ks_12b.dat   3259036   1334963    2639964   1681436    2639964   1602435
7    ks_12c.dat   2489815    926226    1808471   1152681    1808471   1303224
8    ks_12d.dat   3453702   1679959    3406646   1724265    3406646   1858992
9    ks_12e.dat   2520392   1277814    2429214   1216398    2429214   1309915
10   ks_16a.dat   3780355   1654432    3150713   1886539    3150713   2018230
11   ks_16b.dat   4426945   1838356    3601726   2182562    3601726   2170190
12   ks_16c.dat   4323280   1741661    3539978   2125245    3539978   2176322
13   ks_16d.dat   4450938   2051218    4155271   2189910    4155271   2207441
14   ks_16e.dat   3760429   1735397    3442535   1954173    3442535   1967510
15   ks_20a.dat   5169647   2558243    5101533   2658865    5101533   2721946
16   ks_20b.dat   4681373   2230065    4543967   2419141    4543967   2383424
17   ks_20c.dat   5063791   2128763    4361690   2410432    4361690   2723135
18   ks_20d.dat   4286641   1870486    3557405   2158431    3557405   2276327
19   ks_20e.dat   4476000   2115412    4173744   2159969    4173744   2294511
```

20	ks_24a.dat	6404180	2886589	5845661	3174264	5845661	3393387
21	ks_24b.dat	5971071	2961351	5941814	3019080	5941814	3164151
22	ks_24c.dat	5870470	2505304	5008038	2830470	5008038	3045772
23	ks_24d.dat	5762284	2711513	5247821	3047367	5247821	3135427
24	ks_24e.dat	6654569	3278044	6634696	3296337	6634696	3401688

	gr_weight	ga_value	ga_weight
0	1714834	925369	1714834
1	1421763	853704	1750340
2	1422422	771637	1609296
3	1682688	1084704	2059405
4	2377405	1300939	2377405
5	2672179	1468476	2804581
6	2953017	1753926	3254705
7	2406387	1329478	2458307
8	3412958	1858992	3412958
9	2477116	1309915	2477116
10	3768480	2018230	3768480
11	4071350	2311731	4392978
12	4054333	2282303	4315302
13	4245406	2298302	4422372
14	3616049	2030691	3755734
15	5054489	2788040	5161352
16	4471059	2471511	4676284
17	5029940	2723135	5029940
18	4273053	2280911	4275282
19	4353690	2350457	4471547
20	6379172	3393387	6379172
21	5911388	3194906	5970122
22	5820857	3066886	5848030
23	5734259	3150365	5754023
24	6435390	3501861	6649161

In the context of the 25 instances of the 0/1 knapsack problem, the genetic algorithm consistently outperformed the greedy algorithms. Specifically, it achieved solutions that were equivalent to the best greedy algorithm results in 8 cases and surpassed them in 17 cases, with up to 6% improvement.

Resources

- DEAP is an evolutionary computation framework designed for rapid prototyping and testing of ideas. This environment has been developed at Université Laval since (at least) 2012.
  - Solving the knapsack problem using DEAP, complete example.