

Régression logistique entraînée par descente de gradient (par lots)

CSI4506 Introduction à l'intelligence artificielle

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Dans ce cahier, nous développons notre implémentation de la régression logistique, en utilisant la descente de gradient par lots pour l'entraînement. De plus, nous implémentons les fonctions `compute_roc_curve` et `compute_auc`. Par la suite, nous analysons deux jeux de données réalistes mais simples en utilisant les bibliothèques `scikit-learn`.

Logistic Regression

L'implémentation de la régression logistique présentée dans les notes de cours présente certaines limitations pratiques, notamment le besoin pour les utilisateurs d'ajouter manuellement une colonne de uns à la matrice X pour tenir compte du terme d'interception, θ_0 . Une telle exigence peut être considérée comme sous-optimale car elle expose les utilisateurs à des détails d'implémentation inutiles. Pour résoudre ce problème, je propose une implémentation orientée objet qui améliore à la fois la clarté et l'utilisabilité. Cette implémentation utilise efficacement la bibliothèque [NumPy](#).

```
import numpy as np

class LogisticRegression:

    """
    A didactic implementation of binary logistic regression trained with
    (batch) gradient descent.

    Parameters
    -----
    learning_rate : float, default=0.1
```

```

        Step size for gradient descent.
max_iter : int, default=1000
        Number of gradient descent iterations.
random_state : int, default=42
        Seed for reproducible initialization of parameters.

Notes
-----
- This implementation expects binary labels {0, 1}.
- The intercept (bias) term is added internally during `fit`.
- For simplicity, there is no regularization and no early stopping.
"""

def __init__(self, learning_rate: float = 0.1, max_iter: int = 1000,
random_state: int = 42):
    self.learning_rate = learning_rate
    self.max_iter = max_iter
    self.random_state = random_state

    # Attributes set after fitting
    self._theta = None          # shape: (n_features + 1,) including intercept
    self._loss_history = None   # list of floats
    self._n_features = None     # number of features seen in X during fit (without intercept)
    self._fitted = False

# ----- Public API -----

def fit(self, X: np.ndarray, y: np.ndarray) -> "LogisticRegression":
    """
    Fit the model parameters using gradient descent.

    Parameters
    -----
    X : array-like, shape (n_samples, n_features)
        Feature matrix (NO intercept column; it will be added internally).
    y : array-like, shape (n_samples,)
        Binary target labels in {0, 1}.

    Returns
    -----
    self : LogisticRegression
    """

```

```

X = self._as_2d_array(X, name="X")
y = self._as_1d_array(y, name="y")
self._check_binary_labels(y)

m, n = X.shape
self._n_features = n

# Add intercept column of ones
Xb = self._add_intercept(X)

# Reproducible small random init (didactic: shows that init can matter)
rng = np.random.default_rng(self.random_state)
self._theta = rng.normal(loc=0.0, scale=1e-2, size=n + 1)

self._loss_history = []

# Gradient descent
for _ in range(self.max_iter):
    z = Xb @ self._theta
    h = self._sigmoid(z)                # predicted probabilities
    grad = (Xb.T @ (h - y)) / m        # gradient of binary cross-entropy
    self._theta -= self.learning_rate * grad  # update
    self._loss_history.append(self._bce_loss(h, y))

self._fitted = True
return self

def predict_proba(self, X: np.ndarray) -> np.ndarray:
    """
    Return predicted probabilities for the positive class.

    Raises
    -----
    RuntimeError : if called before fit
    ValueError : if X has a different number of features than seen in fit
    """
    self._ensure_fitted()
    X = self._as_2d_array(X, name="X")
    self._ensure_same_n_features(X)

    Xb = self._add_intercept(X)
    return self._sigmoid(Xb @ self._theta)

```

```

def predict(self, X: np.ndarray, threshold: float = 0.5) -> np.ndarray:
    """
    Return class predictions (0/1) using a probability threshold.

    Raises
    -----
    RuntimeError : if called before fit
    ValueError : if X has a different number of features than seen in fit
    """
    proba = self.predict_proba(X)
    return (proba >= threshold).astype(int)

def get_loss_history(self) -> list:
    """
    Return a copy of the loss (binary cross-entropy) history collected during fit.

    Raises
    -----
    RuntimeError : if called before fit
    """
    self._ensure_fitted()
    return list(self._loss_history)

# ----- Internal helpers -----

@staticmethod
def _sigmoid(z: np.ndarray) -> np.ndarray:
    # Numerically stable sigmoid
    # For large negative/positive z, np.exp is still fine here; clip to avoid log(0) later
    return 1.0 / (1.0 + np.exp(-z))

@staticmethod
def _bce_loss(h: np.ndarray, y: np.ndarray) -> float:
    # Binary cross-entropy with epsilon for numerical stability
    eps = 1e-12
    h_clipped = np.clip(h, eps, 1.0 - eps)
    return float(-np.mean(y * np.log(h_clipped) + (1 - y) * np.log(1 - h_clipped)))

@staticmethod
def _as_2d_array(X, name="X") -> np.ndarray:
    X = np.asarray(X, dtype=float)
    if X.ndim != 2:

```

```

        raise ValueError(f"{name} must be a 2D array of shape (n_samples, n_features).")
    return X

    @staticmethod
    def _as_1d_array(y, name="y") -> np.ndarray:
        y = np.asarray(y, dtype=float)
        if y.ndim != 1:
            raise ValueError(f"{name} must be a 1D array of shape (n_samples,).")
        return y

    @staticmethod
    def _check_binary_labels(y: np.ndarray) -> None:
        values = np.unique(y)
        if not np.array_equal(values, [0, 1]) and not np.array_equal(values, [0.0, 1.0]):
            raise ValueError("y must contain only binary labels {0, 1}.")

    def _add_intercept(self, X: np.ndarray) -> np.ndarray:
        ones = np.ones((X.shape[0], 1), dtype=float)
        return np.hstack([ones, X])

    def _ensure_fitted(self) -> None:
        if not self._fitted or self._theta is None:
            raise RuntimeError("This LogisticRegression instance is not fitted yet. Call `fit` to fit the model.")

    def _ensure_same_n_features(self, X: np.ndarray) -> None:
        if X.shape[1] != self._n_features:
            raise ValueError(
                f"Feature mismatch: X has {X.shape[1]} features, but the model was fitted with {self._n_features} features."
            )

```

Exemple 1

Étape 0 : Importer les bibliothèques, définir les variables globales

```

import matplotlib.pyplot as plt
from sklearn.datasets import make_blobs
from sklearn.datasets import load_breast_cancer
from sklearn.datasets import fetch_openml
from sklearn.metrics import classification_report, confusion_matrix, ConfusionMatrixDisplay

```

```
from sklearn.metrics import roc_curve, roc_auc_score
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression as SKLogisticRegression

seed=42
```

Étape 1 : Créer un jeu de données simplifié

```
X, y = make_blobs(n_samples=1000, n_features=2, centers=2, cluster_std=2.5, random_state=42)

# Split into train and test sets
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.3, random_state=42
)
```

Visualisation de l'ensemble d'entraînement.

```
plt.figure(figsize=(7, 6))

plt.scatter(
    X_train[y_train == 0, 0], X_train[y_train == 0, 1],
    color="tab:blue", marker="o", edgecolor="k", alpha=0.8, label="Class 0 (train)"
)
plt.scatter(
    X_train[y_train == 1, 0], X_train[y_train == 1, 1],
    color="tab:orange", marker="o", edgecolor="k", alpha=0.8, label="Class 1 (train)"
)

plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.title("Training Data Before Logistic Regression")
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```



Étape 2 : Entraîner notre classe LogisticRegression

```
model = LogisticRegression(learning_rate=0.1, max_iter=500, random_state=42)
model.fit(X_train, y_train)
```

```
<__main__.LogisticRegression at 0x16dbec980>
```

```
# --- Step 3: Predictions and classification report ---
y_pred = model.predict(X_test)
print("Classification Report:\n")
print(classification_report(y_test, y_pred))
```

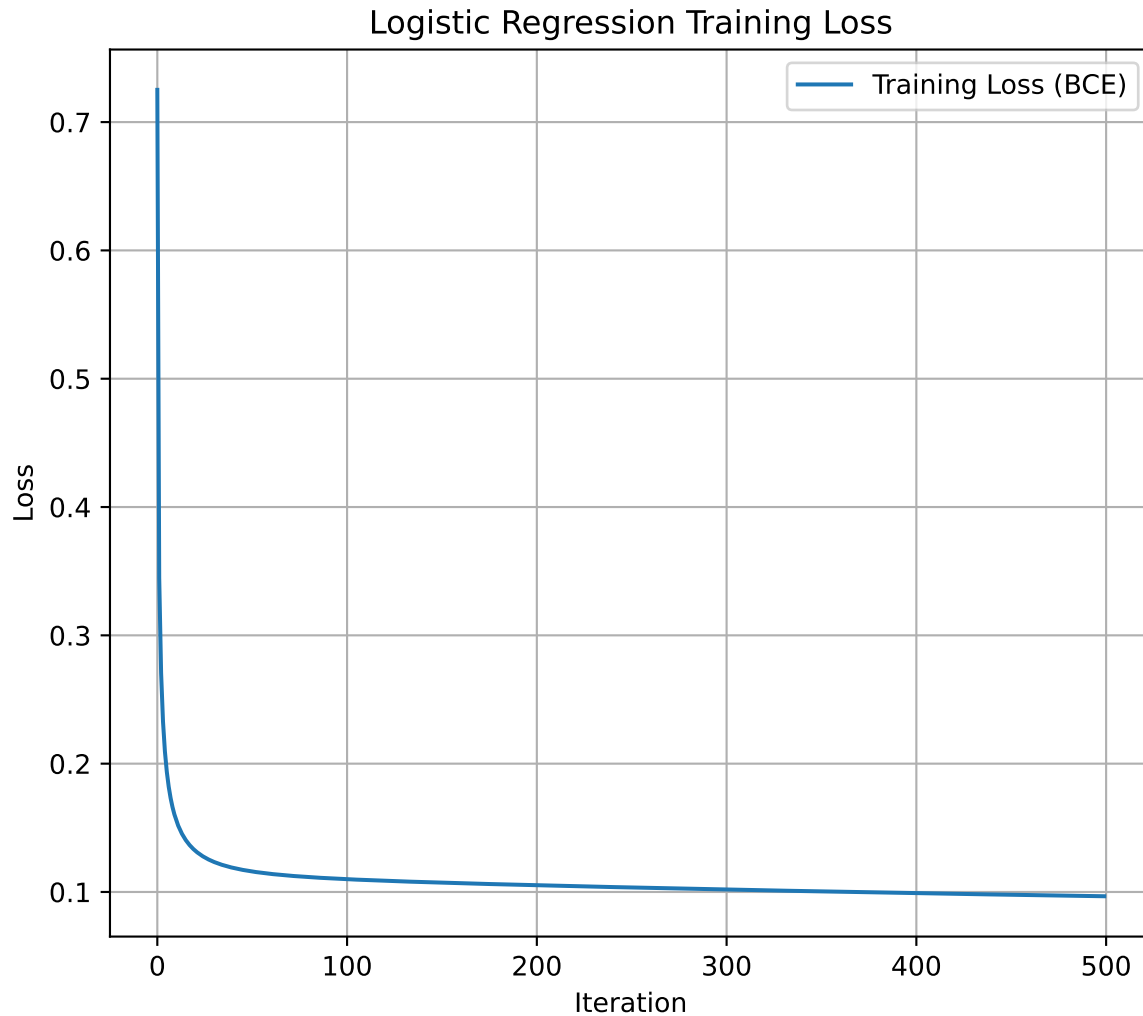
Classification Report:

	precision	recall	f1-score	support
0	0.98	0.99	0.99	151
1	0.99	0.98	0.99	149
accuracy			0.99	300
macro avg	0.99	0.99	0.99	300
weighted avg	0.99	0.99	0.99	300

```
# --- Step 4: Plot the loss history ---

losses = model.get_loss_history()

plt.figure(figsize=(7, 6))
plt.plot(losses, label="Training Loss (BCE)")
plt.xlabel("Iteration")
plt.ylabel("Loss")
plt.title("Logistic Regression Training Loss")
plt.legend()
plt.grid(True)
plt.show()
```



```
# --- Step 5: Decision boundary (2D) ---  
# Works because we used n_features=2 above.  
# If you use more than 2 features, select two columns to visualize (e.g., X[:, :2]).  
  
def plot_decision_boundary(model, X_train, y_train, X_test, y_test, h=0.02):  
    """  
    Plot probability heatmap, 0.5 decision contour, and data points.  
  
    Parameters  
    -----  
    model : fitted LogisticRegression (our class)  
    X_train, y_train : training data (2D features)
```

```

X_test, y_test    : test data (2D features)
h : float, grid step size (smaller -> finer mesh)
"""

# 1) Build a mesh over the feature space (with padding for nicer margins)
x_min = min(X_train[:, 0].min(), X_test[:, 0].min()) - 0.5
x_max = max(X_train[:, 0].max(), X_test[:, 0].max()) + 0.5
y_min = min(X_train[:, 1].min(), X_test[:, 1].min()) - 0.5
y_max = max(X_train[:, 1].max(), X_test[:, 1].max()) + 0.5

xx, yy = np.meshgrid(
    np.arange(x_min, x_max, h),
    np.arange(y_min, y_max, h)
)
grid = np.c_[xx.ravel(), yy.ravel()]

# 2) Predict probabilities on the grid (class 1)
Z = model.predict_proba(grid).reshape(xx.shape)

# 3) Plot
plt.figure(figsize=(7, 6))

# Heatmap of  $P(y=1|x)$ ; levels=25 for smooth look
cntr = plt.contourf(xx, yy, Z, levels=25, alpha=0.8)
cbar = plt.colorbar(cntr)
cbar.set_label("P(class = 1)")

# Decision boundary at probability 0.5
plt.contour(xx, yy, Z, levels=[0.5], linewidths=2)

# Training points
plt.scatter(
    X_train[y_train == 0, 0], X_train[y_train == 0, 1],
    marker="o", edgecolor="k", alpha=0.9, label="Train: class 0"
)
plt.scatter(
    X_train[y_train == 1, 0], X_train[y_train == 1, 1],
    marker="o", edgecolor="k", alpha=0.9, label="Train: class 1"
)

# Test points (different marker)
plt.scatter(
    X_test[y_test == 0, 0], X_test[y_test == 0, 1],

```

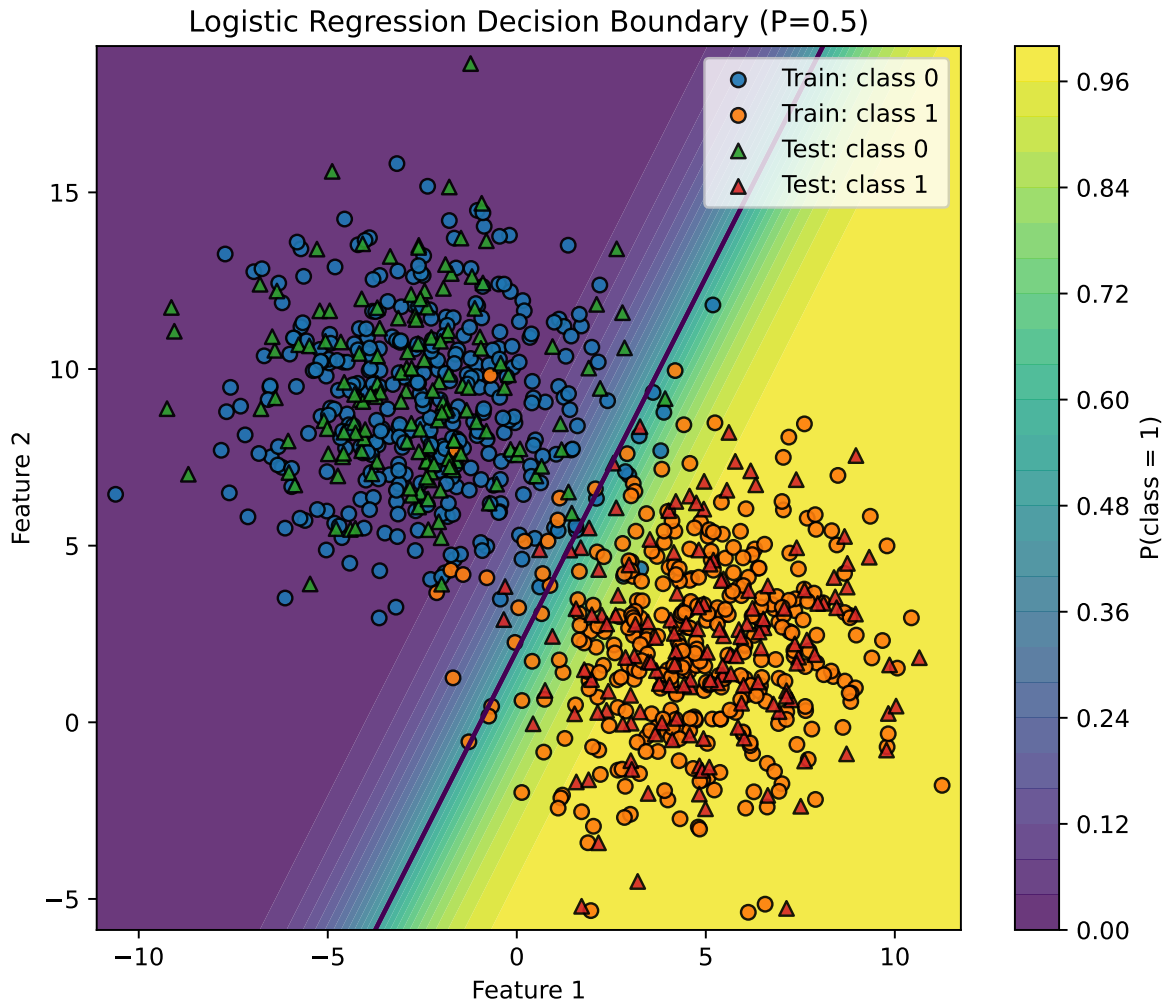
```

        marker="^", edgecolor="k", alpha=0.9, label="Test: class 0"
    )
    plt.scatter(
        X_test[y_test == 1, 0], X_test[y_test == 1, 1],
        marker="^", edgecolor="k", alpha=0.9, label="Test: class 1"
    )

    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.title("Logistic Regression Decision Boundary (P=0.5)")
    plt.legend(loc="best", frameon=True)
    plt.tight_layout()
    plt.show()

plot_decision_boundary(model, X_train, y_train, X_test, y_test)

```



Implémentation de la courbe ROC/AUC

```
def compute_roc_curve(y_true, y_scores, thresholds):
    tpr_list, fpr_list = [], []
    for thresh in thresholds:
        # Classify as positive if predicted probability >= threshold
        y_pred = (y_scores >= thresh).astype(int)
        TP = np.sum((y_true == 1) & (y_pred == 1))
        FN = np.sum((y_true == 1) & (y_pred == 0))
        FP = np.sum((y_true == 0) & (y_pred == 1))
```

```

    TN = np.sum((y_true == 0) & (y_pred == 0))
    TPR = TP / (TP + FN) if (TP + FN) > 0 else 0
    FPR = FP / (FP + TN) if (FP + TN) > 0 else 0
    tpr_list.append(TPR)
    fpr_list.append(FPR)

    tpr_list.reverse()
    fpr_list.reverse()

    return np.array(fpr_list), np.array(tpr_list)

```

Implémenter AUC ROC

```

def compute_auc(fpr, tpr):
    """
    Compute the Area Under the Curve (AUC) using the trapezoidal rule.

    fpr: array of false positive rates
    tpr: array of true positive rates
    """
    return np.trapezoid(tpr, fpr)

```

Exemple : Graphique

```

# Compute predicted probabilities for the positive class on the test set
y_probs = model.predict_proba(X_test)

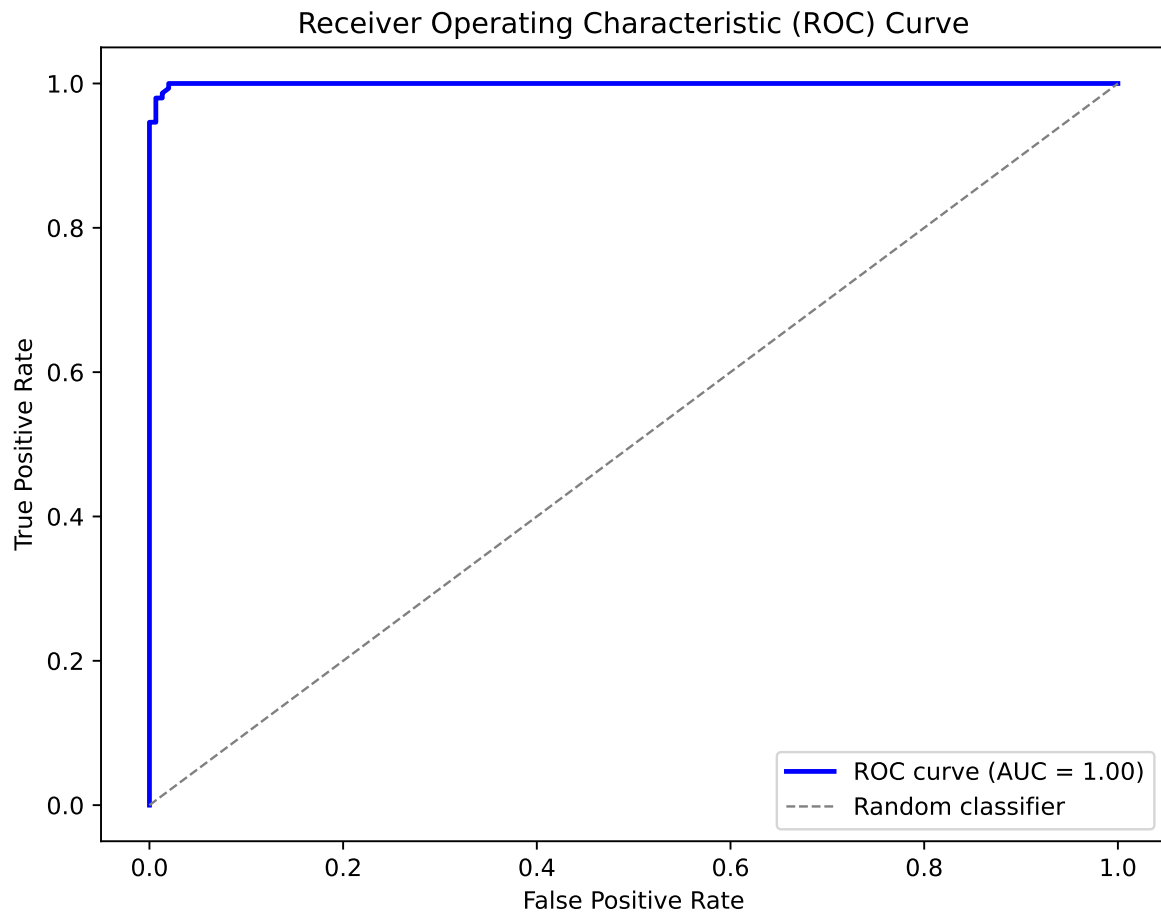
# Define a set of threshold values between 0 and 1 (e.g., 100 equally spaced thresholds)
thresholds = np.linspace(0, 1, 100)

# Compute the ROC curve (FPR and TPR for each threshold)
fpr, tpr = compute_roc_curve(y_test, y_probs, thresholds)
auc_value = compute_auc(fpr, tpr)

# Plot the ROC curve
plt.figure(figsize=(8, 6))
plt.plot(fpr, tpr, color='blue', lw=2, label='ROC curve (AUC = %0.2f)' % auc_value)
plt.plot([0, 1], [0, 1], color='gray', lw=1, linestyle='--', label='Random classifier')

```

```
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver Operating Characteristic (ROC) Curve')
plt.legend(loc="lower right")
plt.show()
```



Exemple 2 (cancer du sein)

Dans cet exemple, nous utilisons la régression logistique de la bibliothèque scikit-learn, ainsi que l'ensemble de données sur le cancer du sein, également disponible via scikit-learn. Il est conseillé de modifier le code fourni pour évaluer si le modèle précédemment introduit atteint

des performances comparables. Comme on peut le constater, dans cet exemple, les classes sont facilement séparables par une frontière de décision linéaire.

```
# Goal: Classify tumors as malignant (1) or benign (0)
# Dataset: sklearn.datasets.load_breast_cancer
# Model: Logistic Regression (scikit-learn)

# 1. Load dataset
data = load_breast_cancer()
X, y = data.data, data.target
print(f"Dataset shape: {X.shape}, Labels: {np.bincount(y)}")
print("Feature names (first 5):", data.feature_names[:5])

# 2. Train/test split
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.2, random_state=42, stratify=y
)

# 3. Scale features (important for gradient-based methods)
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# 4. Train Logistic Regression
clf = SKLogisticRegression(max_iter=5000, random_state=42)
clf.fit(X_train_scaled, y_train)

# 5. Evaluate
y_pred = clf.predict(X_test_scaled)

print("\nClassification Report:")
print(classification_report(y_test, y_pred, target_names=data.target_names))

# Confusion matrix
cm = confusion_matrix(y_test, y_pred, labels=clf.classes_)
disp = ConfusionMatrixDisplay(confusion_matrix=cm, display_labels=data.target_names)
disp.plot(cmap="Blues")
plt.show()

# 6. Inspect model coefficients
# (Shows how each feature contributes to decision boundary)
coef = clf.coef_[0]
feature_importance = sorted(zip(data.feature_names, coef), key=lambda x: abs(x[1]), reverse=)
```

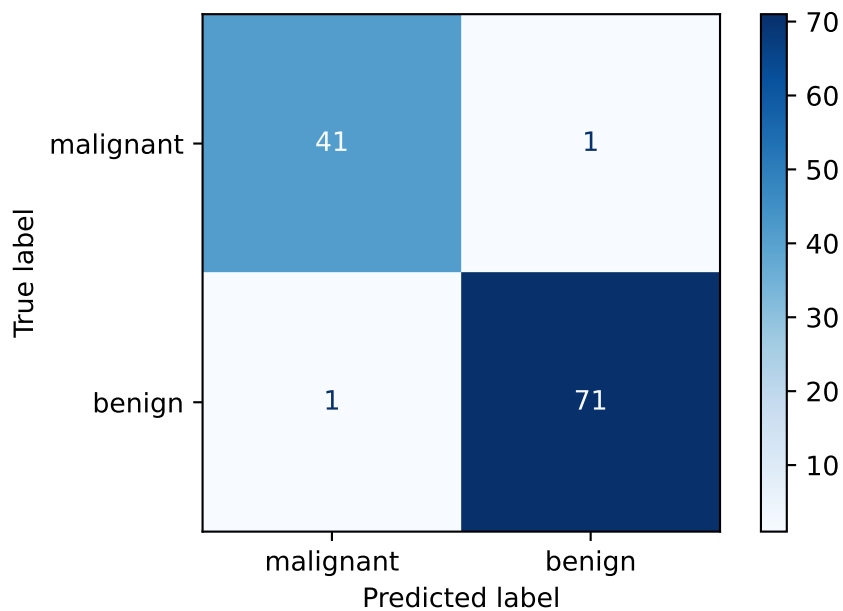
```
print("\nTop 5 influential features:")
for name, weight in feature_importance[:5]:
    print(f"{name:25s} {weight:.3f}")
```

Dataset shape: (569, 30), Labels: [212 357]

Feature names (first 5): ['mean radius' 'mean texture' 'mean perimeter' 'mean area' 'mean smoothness']

Classification Report:

	precision	recall	f1-score	support
malignant	0.98	0.98	0.98	42
benign	0.99	0.99	0.99	72
accuracy			0.98	114
macro avg	0.98	0.98	0.98	114
weighted avg	0.98	0.98	0.98	114



Top 5 influential features:

worst texture	-1.255
radius error	-1.083

worst concave points	-0.954
worst area	-0.948
worst radius	-0.948

Exemple 3 (Diabète chez les Indiens Pima)

Dans ce troisième exemple, la régression logistique est confrontée à un défi plus important. Atteindre un taux de vrais positifs (TPR) ou rappel de 85 % nécessite un compromis significatif, entraînant un taux de faux positifs (FPR) où 31 % des instances réellement négatives sont incorrectement classées comme positives.

```
# ROC demo: Logistic Regression on Pima Indians Diabetes
# -----
# - Fetches the Pima dataset from OpenML
# - Trains a SKLogisticRegression
# - Plots ROC curve and reports AUC

import warnings
warnings.filterwarnings("ignore")

# 1) Load Pima from OpenML (try a few common names to be robust across mirrors)
candidates = ["diabetes", "pima-indians-diabetes", "diabetes_binary", "diabetes_numeric"]
X, y, feat_names = None, None, None

for name in candidates:
    try:
        ds = fetch_openml(name=name, as_frame=True)
        df = ds.frame.copy()
        # Identify target column candidates
        for target_col in ["class", "Outcome", "diabetes", "target"]:
            if target_col in df.columns:
                y = df[target_col]
                X = df.drop(columns=[target_col])
                feat_names = X.columns.tolist()
                # Ensure binary labels {0,1}
                if y.dtype.kind in "OUS":
                    y = y.astype(str).str.lower().replace({
                        "tested_positive": 1, "tested_negative": 0,
                        "pos": 1, "neg": 0, "positive": 1, "negative": 0,
                        "yes": 1, "no": 0
                    })
    }
```

```

        y = y.astype(int)
        break
    if X is not None and len(np.unique(y)) == 2:
        print(f"Loaded OpenML dataset: '{name}' with shape {X.shape}")
        break
except Exception:
    continue

if X is None:
    raise RuntimeError("Could not load the Pima dataset from OpenML. "
                       "Check your internet connection or try again later.")

# 2) Train/test split (stratified for class balance)
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.25, random_state=42, stratify=y
)

# 3) Scale features (helps LR optimization)
scaler = StandardScaler()
X_train_s = scaler.fit_transform(X_train)
X_test_s = scaler.transform(X_test)

# 4) Train Logistic Regression
clf = SKLogisticRegression(max_iter=2000, random_state=42)
clf.fit(X_train_s, y_train)

# 5) ROC curve & AUC
y_score = clf.predict_proba(X_test_s)[: , 1]      # probability of positive class
fpr, tpr, thresholds = roc_curve(y_test, y_score)
auc = roc_auc_score(y_test, y_score)

# 6) Plot
plt.figure(figsize=(5, 5))
plt.plot(fpr, tpr, lw=2, label=f"LogReg (AUC = {auc:.3f})")
plt.plot([0, 1], [0, 1], lw=1, linestyle="--", label="Chance")
plt.xlim(0, 1); plt.ylim(0, 1)
plt.xlabel("False Positive Rate")
plt.ylabel("True Positive Rate")
plt.title("ROC Curve - Pima Indians Diabetes")
plt.legend(loc="lower right")
plt.tight_layout()
plt.show()

```

```

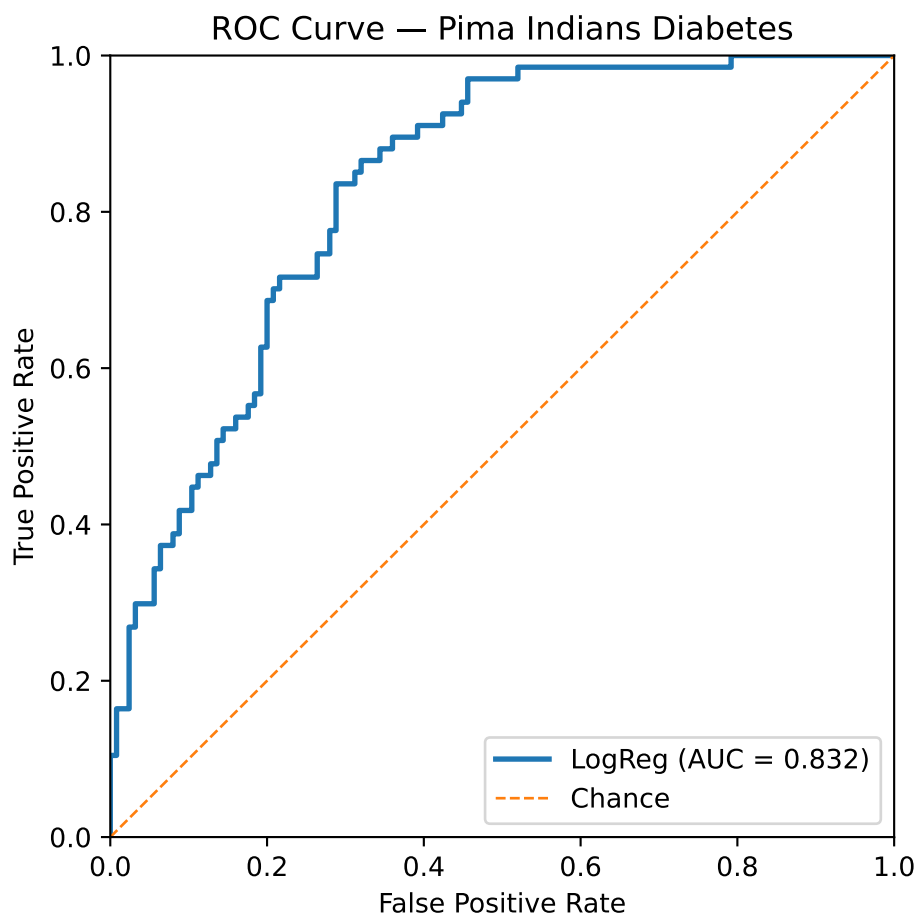
# Target TPR
target_tpr = 0.85

# Find index of TPR closest to target
idx = np.argmin(np.abs(tpr - target_tpr))

print(f"Closest TPR: {tpr[idx]:.3f}")
print(f"Corresponding FPR: {fpr[idx]:.3f}")
print(f"Threshold: {thresholds[idx]:.3f}")

```

Loaded OpenML dataset: 'diabetes' with shape (768, 8)



Closest TPR: 0.851

Corresponding FPR: 0.312
Threshold: 0.271